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Prediction of mortality characteristics of industrial property groups

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PREDICTION OF MORTALITY CHARACTERISTICS

OF INDUSTRIAL PROPERTY GROUPS
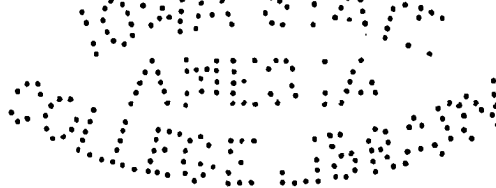
by

Harold Andrews Cowles, Jr.

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TABLE OF CONTENTS

INTRODUCTION

In any prediction of mortality behavior there is an element of fact and an element of judgment. The basic fact is the historic record of the property as experienced under the conditions imposed by such things as economic trends, managerial policies, maintenance procedures, etc. The judgment is principally represented by the extent to which the engineer believes the facts will be duplicated in the future and by the additional conditions he feels are needed to complete the prognosis. In some instances, e.g., on relatively stable property with complete historic records, the facts can be considered a reasonably valid description of the future. In other instances, the historic facts are meager and, hence, cannot be relied upon too heavily. Here, judgment is essentially the only consideration. It would seem to be an acceptable—but not an indisputable—truism that the more complete the factual information, the more realistic the prediction, regardless of the extent to which judgment is required. The present investigation is concerned with the development of these factual data.

While the historic record of a property's retirement experience is factual, it is not particularly useful in its original form. That is, the number of dollars remaining each

year from the 1936 installation of centrifugal gas pumps is
of little help to the engineer trying to decide what mortality
experience to expect from the investment in equipment presently
being installed. Likewise, the records of all the other
years' installations are of little help unless their ex-
perience can be aggregated in some way so that an interpretable
result is obtained.

Consequently, the study of the mortality behavior of
industrial properties has drawn heavily upon the techniques
developed by life insurance actuaries in their presentation
of human life characteristics. In reporting some of the
first analysis work with industrial retirement, Kurtz (27)
pointed out the early recognition of the similarities between
the two areas. Basically, it was seen that by gathering in-
dustrial or similar property units into single accounts,
group behavior could be analyzed and predicted by statistical
procedures in which the retirements were related to the age of
the property. While these results could not be said to be
applicable to a particular unit, they did represent the ex-
pected experience for the group as a whole with a certain
degree of assurance.

Present day analysis techniques as well as the available
data represent considerable improvement over those used in
the earlier attempts, yet, the confidence the engineer may
put in his result is still far less than that which the

actuary is able to assume. The tremendous difference in the amount of experience available for analysis is an obvious factor here.

A more basic consideration, though, is the fact that property units not only "die", i.e., wear out or become inoperative, but they also are "fired". In other words, a machine may be retired because of inadequacy or obsolescence while still in perfectly good running order. A recent report indicates that in some industries as high as 80 per cent of the retirements were due to other than "natural causes" (14). Inadequacy and obsolescence were mentioned as the principal causes for removal from service.

Consequently, management policy and technological change are generally more important in determining the time and the amount of industrial equipment retirement than the property's age or its physical condition. Any behavior summarization based upon the analysis of past experience is bound to be influenced by these previous policies and factors. To use these same mortality representations as a basis for predictions of future behavior would be improper unless the influence of changing policy and of technological developments is fully recognized.

One consideration that might tend to minimize this particular difficulty is that in some instances time is a fairly good indication of obsolescence, e.g., automobiles or

aircraft. It might be that a good relationship between obsolescence and age could be found in a number of other property groups. In such cases, some of the objections to using the so-called age-life approach of the actuary would be eliminated.

Thus, while the adoption of actuarial methods has proved to be extremely helpful, it has not done away with all the problems. Industrial property experience, admittedly, does not meet too well the normal specifications for data subject to statistical analysis. That is, the fundamental "laws" or trends are not consistently maintained due to shifts in such things as equipment replacement policies. Frequently the experience may not cover a long enough period to give a significant measure of any mortality characteristic, or the experience itself may be questionable due to the accounting procedures followed by the firm.

Many of these problems can be handled by the selection of an appropriate analysis procedure as well as a judicious choice of the portions of the experience to analyze. But, these difficulties do tend to emphasize the importance of judgment in interpreting the measures of historic record.

PRESENT INVESTIGATION AND ITS OBJECTIVES

The study reported in this dissertation was undertaken
with the objective of providing information regarding the
reliability and consistency inherent in some of the techniques
used to analyze retirement data. Further, some indication
was desired as to the extent to which the use of these
methods may influence the financial records of a firm.

Numerous analysis procedures have been proposed but two
seem to be used most frequently. They are, first, the use of
the Iowa type curves and, second, the fitting of retirement
ratios by least squares. It was believed that these two
methods represent the most logical choices to be subjected to
a comparative investigation. The desirability of their
selection is also based upon factors other than their
acceptance. Normally, they are found in "opposite camps" in
discussions on appropriate forms of life analysis. That is,
the Iowa type curves represent the graphical, curve matching
approach while the retirement ratios are fitted by the use of
mathematics. Likewise, the Iowa curves are applied directly
to a representation of the observed mortality dispersion
while in an orthogonal analysis the retirement ratios are
first fitted to a mathematical expression which is then used

to develop a smoothed version of the mortality dispersion.

The objectives of this investigation may now be stated more specifically as follows:

1. To determine an indication of whether either the Iowa type curve method or the use of orthogonal polynomials consistently gives better estimates of industrial property mortality patterns.

2. To determine an indication of whether either method gives better results when applied to certain dispersion types, and to data varying in degree of completeness.

3. To determine in particular whether the use of the Iowa type causes any significant bias or error in mortality dispersion estimates made of property retirement data varying in shape, length of stub curve, or average service life.

Since predictions of mortality dispersion are used principally for depreciation considerations, it was desired to represent, or at least interpret, any significant indications in terms of the two phases of depreciation, i.e., the annual accrual and the accrued reserve.

It is therefore believed that any conclusive findings resulting from this investigation will be of value to most depreciation engineers. Those men who work with the Iowa type curves or those who choose to fit retirement ratios may find the information particularly pertinent. But the results should prove of general interest to the engineer employing

any of the other actuarial approaches as well. This is because the divergent philosophies of data analysis present in the two methods considered in this study embrace those ideas upon which most of the currently used procedures are based.

The present investigation is only the first of a series that should be conducted to determine the characteristics of the many life estimating techniques. Those methods designed for firms which do not have complete retirement experience suitable for an actuarial analysis should receive particular attention. The companies using these methods are quite numerous because of the expense necessary to maintain sufficient property records. Admittedly, there are few firms outside of public utility industry that could provide the aged retirement experience needed for the analysis procedures used in this study.

RELATED CONCEPTS AND PROCEDURES

The practitioner will recognize that the question of
mortality characteristics referred to in this investigation
is only one of many that need to be considered by a company's
management--and perhaps a regulatory body--in arriving at a
realistic depreciation policy covering future operations. In
order to put the problems discussed in this dissertation in
their proper perspective, the following discussion of relative
terminology, concepts, and procedures is presented.

Mortality Dispersion

The percentage or number of an original installation
that would be remaining in service as of any age is the
mortality characteristic of an industrial property group which
is of most interest to the depreciation engineer. This basic
trait of the group is known as its mortality dispersion and
it is normally represented either in tabular form as a life
table or graphically as a survivor curve. An alternative
graphical form is the frequency curve which shows the per-
centage or number of units retired at each age. This form is
rarely used, however, since it is not as conveniently derived
from the retirement data as the life table or the survivor

curve.

A measure of the total amount of service to be expected
from the group can be developed directly from the mortality
dispersion by the successive addition of incremental service
contributions. That is, the amount of the service rendered
by a property group over any small time interval would be the
product of the number of units in service multiplied by the
length of the interval. The summation of the service from
all such time intervals extending from installation to the
date the last unit is expected to be retired is the total
service expected. Mathematically, this may be represented by

$$\text{Total expected service} = \int_{n}^{o} f(x)dx$$

where $f(x)$ = expression for per cent or number surviving in
terms of age x

n = maximum age of any unit.

Since $f(x)$ represents the equation of the survivor curve, the
integration determines the area under the curve. Thus, this
area is the basic measure of total expected service from any
installation assuming the property follows the particular dis-
persion pattern.

The amount of service expected may be approximated
satisfactorily by the numerical integration of the life table
values. In most cases these entries are stated as of ages $\frac{1}{2}$,
$1\frac{1}{2}$, $2\frac{1}{2}$, etc. This is because property records are normally

balanced and closed as of the end of the accounting year. On the other hand, all additions and retirements are assumed to be made as of the mid-year date. Thus new units are considered to be one-half year old at the end of the first year, one and half year old at the end of the second, and so on.

With this half-year convention the total service rendered by the units which are retired as of any age x would be (x) $(L_{x-\frac{1}{2}} - L_{x+\frac{1}{2}})$ where L represents units or percentage in service at the ages indicated in the subscripts. Therefore, the total service to be expected from a property group at installation would be given by the following:

$$( )(100\% - L_\frac{1}{2}) + (1)(L_\frac{1}{2} - L_{1\frac{1}{2}}) + (2)(L_{1\frac{1}{2}} - L_{2\frac{1}{2}}) + \ldots$$
$$+ (n-1)(L_{n-3/2} - L_{n-\frac{1}{2}}) + nL_{n-\frac{1}{2}}$$

or,

$$\text{Total expected service} = 25\% + 3/4 L_\frac{1}{2} + L_{1\frac{1}{2}} + L_{2\frac{1}{2}}$$
$$+ \ldots + L_{n-\frac{1}{2}}.$$

Following a similar analysis one may express the expected future service as of any date subsequent to installation as

$$\text{Expected remaining service at age } x = \int_n^x f(x)\, dx$$

in the case of a survivor curve, and for the life table:

$$= L_{x-\frac{1}{2}} + L_{x+\frac{1}{2}} + \text{Remaining life table terms.}$$

The expected service from the average unit at installation or as of some other date is of more interest, usually, than the total service expected from the group. These

measures are merely averages and are indicated in terms of
length of life. Thus at installation the expected life of
the average unit, i.e., the average life of the group, is
found by dividing the total expected service by the number of
units installed or by 100 per cent if percentages are used.
Thus,

$$\text{Average service life, } E = \frac{\text{Total expected service as of age 0}}{100\% \text{ or number of units installed}}$$

It is interesting to note that the total service rendered by
the group behaving according to the expected mortality dis-
persion is the same as that which would be given by the group
had every unit remained in service until the average age was
reached and then all were retired as a group at that time.

The expected life of the average survivor as of some
date other than that of installation is termed the expectancy
of the group. In equation form this becomes

$$\text{Expectancy, } Ex, \text{ age } x =$$

$$\frac{\text{Expected remaining service at age } x}{\text{Per cent or number surviving at age } x}.$$

By themselves these measures of service are of little conse-
quence, but when one considers their impact upon the depre-
ciation policies of an industrial firm, they are seen to be
of considerable importance. Most all of the allocation pro-
cedures in use today relate the annual charge to depreciation
directly to the average service life.[a] Likewise, in many

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[a]Infra, p. 37-43.

cases the adequacy of past depreciation policies as represented by the depreciation reserve can be established by using expectancies.[a]

Consequently, it is extremely desirable to have as complete knowledge of the mortality dispersion as possible when establishing depreciation policy. The major difficulty is that this information is needed at the date of installation or at least while the property is still in service. However, the actual dispersion for a particular group is never known until all the units have been retired. Thus, mortality dispersion is something that must always be predicted for depreciation purposes.

Mortality Dispersion and Service Life Analysis

Some type of analysis of past experience is a desirable preliminary to the effective prediction of future mortality behavior. While it is generally recognized that past experience may not be a good indicator of future happenings, it is believed that the trends shown by even sparse data are better than guesses or hunches in this initial phase.

The various analysis procedures available have been classified as actuarial, turnover, and forecast (2). Each general category can be distinguished from the others by the amount of information required. The actuarial methods, for

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[a]_Infra_, p. 46.

example, need the age and amount of all survivors additions, and retirements while the turnover techniques can be performed with only the amounts of the retirements and survivors for each year. Frequently, the information required by even the turnover methods are not obtainable. In situations of this sort the only basis for a mortality prediction is judgement based upon personal experience and a study of factors related to retirement behavior. This is the essence of the forecast method.

Any of the techniques generally considered to fall under the above categories will produce an indication of service life. However, with one exception, only the actuarial approaches will result in a measure of mortality dispersion. The exception is the simulated plant-record method which, while not functionally similar to the turnover group, is sometimes classified with them because the data requirements are the same. Conceivably, it would be possible to arrive at a predicted dispersion under the forecast method by some subjective means, such as a free hand survivor curve. A much more likely assumption would be that the analyst using the forecast method would refer to a series of actuarial analyses in an effort to discover one which, in his opinion, represented property sufficiently similar to that with which he was concerned. The same approach would have to be adopted when using most turnover methods.

Thus, if complete information is desired concerning the behavior of the property, it appears that the actuarial approaches are to be preferred. The turnover procedures do have a considerable appeal in that they require less involved calculations and place less stringent specifications upon the retirement data. However, if there is a choice between the two approaches, the actuarial seems to be more desirable (2).

The principal techniques in the various analysis categories referred to have been described at length (2, 20, 30, 32, 48). Therefore, they are presented only summarily at this time.

Actuarial

Any statistical summarization or prediction with regard to mortality behavior is, obviously, no more representative than the retirement data subjected to the analysis. Thus, accurate and consistent property accounting is the key to meaningful retirement analyses. Unfortunately, most plant records do not meet these qualifications and so they frequently have to be adjusted in some way to compensate for discrepancies, omissions, or errors caused by changes in classification of accounts or in accounting practice, etc. There are not many literature sources that provide help in handling these problems but the Edison Electric Institute has

presented some specific suggestions in two of their bulletins
(2, 14).

Once the data are deemed appropriate, they are normally
analyzed by one of two approaches. The first is called
"original group". A succession of survivor ratios are
determined by noting the percentage of an initial installa-
tion still in service at each age. It is recognized that
this series of percentages is an expression of the mortality
dispersion directly. The initial installation may be a
single year's additions or those from a group or band of
years. This method is particularly useful in denoting any
characteristic change in mortality behavior between vintages
installations.

The second approach, "retirement rate" or "annual rate"
is much more widely used. In this procedure a series of sur-
vival probabilities or ratios are found for each age interval
by noting the retirement behavior of all the units of that
particular age during a specified experience band of years.
These results are quite significant because all units cur-
rently in service contribute experience to the determination
of one or more survival ratios, depending upon the number of
years included in the experience band. It might be said that
a cross-section of the entire account's behavior was sampled
during a representative time interval. Careful selection of
this experience band is quite desirable since if conditions

prevailing over this period are judged to be similar to what is expected in the future, the subsequent modification of the results for a mortality prediction is minimized. Theoretically, an instantaneous observation period is the ideal experience interval since this would measure the most recent past, reflecting current policies and conditions. However, in an actual analysis even bands of two years in width are not normally recommended for fear of encountering extenuating circumstances which would not be representative and which could distort the results. Marston, Winfrey and Hempstead (30, p. 154) suggest a minimum of three years.

The survivor rates obtained from an original group analysis constitute the life table and a graphical representation of these values produces an unsmoothed survivor curve. If all the units of the initial installation were not retired during the period of the study, the values for per cent surviving would not go to zero and corresponding plot would be a stub curve.

The data from the retirement rate analysis do not represent a life table in their initial state. To make the conversion the per cent surviving at age zero, i.e., 100 per cent, is multiplied by the survivor ratio for the first interval. This determines the percentage still in service at the end of the first interval or the beginning of the second. Likewise, this figure is multiplied by the survival ratio

over the second interval and so on. The successive multipli-
cation is continued until the life table is complete or all
the survival ratios have been used. The graphical presenta-
tion of these data is an unsmoothed survivor curve that may
or may not be stubbed. The next step in the case of either
analysis procedure is to smooth the data and to extrapolate
them where needed to complete the life table.

An alternate approach is available in the case of the
retirement rate method. The analyst may prefer to smooth and
extend the survival (or retirement) ratios themselves before
developing a survivor curve. The proponents of this second
approach argue that:

> (The techniques based upon smoothing a life table
> or survivor curve) are open to the serious objec-
> tion that the manipulative treatment of the data
> by the successive multiplication of "observed"
> survival ratios to obtain an "observed" life
> table, before the fitting process can be begun,
> destroys to a large extent the independence of
> the individual observations. (33, p. 79)

The process of smoothing or fitting survivor data has
received considerable attention and, as a result, numerous
procedures have been proposed. They are based upon the
premise that each property has some "fundamental law" of re-
tirement which must be recognized from the stub data. This
"law" or trend forms the basis for the smoothing and the data
extrapolation.

For the purpose of this discussion these methods have

been classified according to whether the life table or the
survival ratios are smoothed.

Life tables. Two general procedures are used to fit
life tables. One is to use statistical means to express the
available data with a mathematical equation. The other is to
plot the data in the form of a survivor curve and fit it by a
comparison to standard curves. The former technique has been
used by the Bell System for many years. The valuation
engineers of this company prefer to fit the observed life
table to the Gompertz-Makeham equation (20):

$$L_x = K \ S^x \ g^{c^x}$$

where L_x = per cent surviving at age x

K, S, g, c = constants determined from the observed
data.

The engineers of the California Public Utility Commission
recommend the use of the Gompertz equation which is simply:
(46)

$$L_x = K_g \ g^{c^x}$$

when the symbols are the same as those used above. The solu-
tion of this last equation from the observed data is given by
Mills (31). Brennan has done considerable work with this
method of curve fitting (6). Probably his most interesting
proposal dealt with the determination of the unknown in the
equation in the use of a power series (7).

The use of standard curves involves considerably less

calculation time but may not be quite as objective as the
mathematical approach. The standard curves as developed by
the work of Kurtz and Winfrey (27, 48, 51) at the Iowa
Engineering Experiment Station are the most widely recognized.
These so-called Iowa type curves represent standardized mor-
tality frequency curves based upon the analysis of actual in-
dustrial experience. The final set, as issued by the
Station, contained 18 curves which were classified according
to the location of the mode of the frequency curve with res-
pect to the average life as well as the height of the maximum
ordinates.

Since a life table or survivor curve is the most common
and convenient way to represent mortality experience, the
standard frequency curves were each integrated to produce the
corresponding survivor curves for average lives ranging from
5 to 50 years in five-year intervals. The resulting curves
for each type were then plotted on a single sheet of paper to
a standard scale.

The accepted procedure in the use of the curves is to
plot the observed life table to the standard scale on a sheet
of transparent graph paper. This unsmoothed curve is then
laid over each of the type curve sheets. Normally, the type
and average life which best fit the data are determined by
eye with weighting of the individual points done by judgement.
However, a sum of the least squares of the deviations has

been suggested by the Depreciation Accounting Committee of the Edison Electric Institute (14, p. 17).

Kimball proposed a system of standard curves based upon a truncated normal curve (25). His purpose was to develop a series which responded to mathematical analysis much more readily than the Iowa types.

The increasing use of stored program electronic computors has lead Caunt to suggest a series of curves based on the probability function (11). Variation in curve shapes was accomplished by expanding and shrinking the abscissas to the right of the mode, and by changing the starting and stopping point of the curve.

The so-called Patterson series of standard curves appeared in the 1938 Report of the Committee on Depreciation, National Association of Railroad and Utility Commissioners (33). Jeming (22) also proposed a series of standard curves in which the retirement ratios would be represented by

$$R_x = ax^n$$

where R_x = retirement at age x

a,n = constants determined from the data.

None of the above systems, however, have been developed or recognized to the extent that the Iowa types have.

Retirement ratios. The simple assumption behind this approach is that the older the property becomes the more likely it is to be retired. Consequently, retirement ratios

are normally expected to increase with age. The trend of
this relationship is established by fitting the observed data
with either a straight or curved line.

Since the values for the ratios are essentially inde-
pendent, the least squares concept of fitting is generally
used. The most elegant application of this approach was per-
fected by Fisher (18) and was suggested for the analysis of
retirement experience by Mr. Beverly Benson of the New York
Public Service Commission (33). The form of the final equa-
tion relating the retirement ratios or (survivor ratios) to
age would be

$$R = a + bx + cx^2 + dx^3 . . .$$

where R = retirement ratio at an age x

    a, b, c, etc. = constants determined from the data.
Mathematically, the procedure makes use of the orthogonal
polynomials of Tchebychef and, hence, the technique is com-
monly referred to as orthogonal polynomial fitting.

The principal advantage of this method is that the
analyst is able to start with a first degree equation and
evaluate how well the computed values agree with the original
data. If the results are not satisfactory he can develop
the second degree expression from the first by merely com-
puting an additional term. In a similar manner one is able
to go on to the third and the fourth degrees, etc. Experience
has shown that the smoothing of retirement data rarely

requires an equation above the third degree. The use of the various equations have been summarized by Benson as follows:

> The straight line is occasionally the best;
> the second degree is usually the best; the third
> degree fits a number of kinds of property which
> show rapidly decreasing probability of survival
> at early ages, then a slower decrease for some
> years, and then a resumption of rapid decrease;
> there appears to be little justification for
> ever using equations of higher degree than the
> third. (33, p. 79-80)

A second advantage gained in the use of this approach is that the smoothed values may be determined directly without having to solve the appropriate equation for each term. A series of simple summing operations which yield all the points on the smoothed retirement ratio curve has been developed and may be used when a complete life table is desired.

No weighting of the data is possible in this method other than the omission of the retirement ratio values based on the experience of the older units. If these values appear to be erratic and deviate from the trend established by the younger units, they may be ignored. Some judgement needs to be exercised at this stage however.

The ratios may also be fitted to the same equation form by the conventional least square methods as described in a 1942 publication of the Edison Electric Institute (2). In these particular methods it is possible to weight each point with the amount of property in service at each age. However,

unless the weighting question has been deemed critical, the orthogonal solution is most frequently used.

Turnover

The basic principle of the turnover methods is relatively simple. It is merely that in any static but continuing property, i.e., one in which there are additions which essentially replace the retirements, the number of years required to replace completely the plant, one turnover period, is equal to the average life. These studies are normally carried out on a dollar basis so that what is actually "turned over" is the plant investment. Thus, the basic data required are the additions, the retirements and the total plant in service in dollars as of each year. What was said above concerning the reliability of the property records must also be emphasized for these methods. No analysis techniques will correct faulty data.

The turnover period may be determined by plotting the cumulative gross additions and cumulative retirements on the same chart. The horizontal distance between a value on the additions curve and one on the retirements curve represents the turnover period for those units retired as of the date considered on the retirements plot. Another procedure would be to accumulate retirements back from some date until the total would equal the plant balance at an earlier date. The

elapsed time interval would be the turnover period. Still
another approach would be to accumulate gross additions back
from any date till they equalled the balance in the account
as of that date. The time difference again is the turnover
period.

Jeynes (24) and Jeming (22) have presented modifications
of the original turnover methods. Each of these techniques
has considerably more mathematical foundation than the cumu-
lative methods described above.

A very unique analysis technique called the simulated
plant-record method has been proposed by Bauhan (4). This
method merely requires the data needed for a turnover analysis
yet it solves for both the average life and the mortality
dispersion. The analysis is accomplished by first assuming
some known mortality pattern such as one of the Iowa type
curves. Next an estimate of the total dollars in service at
any date is made by multiplying the cost of each year's
additions by the successive percentages surviving at each age
as given by the assumed mortality dispersion. The sums from
each year's installations which are indicated as being in
service during a specified year are added and the resulting
represents a predicted plant balance for that year. A summa-
tion is accomplished for each year and the values are com-
pared with the actual account balances. This whole procedure
is normally repeated for 20 or 30 mortality approximations.

The behavior pattern giving the least sum of squares of deviations from the actual plant balances is assumed to be the best description of the mortality dispersion.

Forecast

In many cases the data necessary for the methods described above are not available. Either the records are not complete or the units are so few and long-lived that little or no experience is obtainable. In instances such as these, the forecast method is the only possible procedure to use for the estimation of service life. This particular approach involves judgment based upon experience, analysis of related economic and policy trends, and any other factors deemed relevant.

Summary

It should be emphasized that every analysis procedure will indicate a slightly different estimate of average service life or mortality dispersion. Likewise, any particular method will also show varying results for the same basic property if different experience periods are considered. The careful analyst must be aware of these factors and try to consider all the reasonable solutions before he finally exercises his judgment to choose the most representative answer.

Depreciation

When approaching the discussion of depreciation, one is likely to experience some trepidation for as Kuhn has indicated:

> Throughout the entire history of the public utility industry there seems to have been confusion, fear, and misunderstanding on the subject of depreciation. (26, p. 199)

However, a closer look at the problem discloses that to a great extent the confusion is due to the failure to delineate the various concepts of depreciation. Bonbright has very carefully specified what he terms the "four basic concepts" as (a) impaired serviceability, (b) fall in value, (c) difference in value present value and present replacement cost, and (d) amortized cost (5). Fitch agreed in essence but preferred "physical condition, value-depreciation, and cost-depreciation", indicating that (c) above was "actually a combination of the cost and value concepts". (19)

Thus, the term depreciation may describe the state of physical deterioration of property, the difference in value or monetary equivalent of an asset as of two different dates (negative as well as positive differences possible), or the extent to which the cost of a piece of equipment has been allocated to production costs. With these possible interpretations in mind, the problem of determining an acceptable

definition is somewhat clarified.

Definition

Probably the most quoted definition of depreciation appeared in a United States Supreme Court decision of 1934. It read as follows:

> Broadly speaking, depreciation is the loss, not restored by current maintenance, which is due to all the factors causing the ultimate retirement of the property. These factors embrace wear and tear, decay, inadequacy, and obsolescence. (43, p. 167)

This definition has been criticised for the failure to specify what it is that is lost through depreciation (30). Federal agencies, in general, have patterned their definitions after the one quoted above and in many cases they have added the terms "value" or "cost" to indicate that which was lost because of depreciation (13). The inclusion of either of these terms does not make the expression completely satisfactory because it then becomes limited to the specific condition, i.e., value depreciation or cost depreciation. Any definition ought to be sufficiently general to embrace all the concepts of depreciation.

After an exhaustive study Fitch (19) proposed the following terminology which seems to be usable and acceptable:

> Depreciation is the decrease in the number of available units of service which a unit of property or group of property units can be expected to render.

Cost-depreciation is the decrease in the
available units of service expressed as a func-
tion of the cost of the property.

Value-depreciation is the change in the
present worth of the anticipated returns from
the services rendered by a property. Value-
depreciation can be determined only after a
valuation is completed and cannot be a factor
of the value of a property.

The significant feature of these definitions is the

specification of service rather than some monetary designa-

tion as the basic measure. This permits a perfectly general

expression for the term, depreciation, and also allows

appropriate sub-definitions by merely relating service to the

desired concept, i.e., cost, value, or physical condition.

Cost-depreciation

The proposition that manufacturing costs rightfully in-

clude some increment of equipment investment representing the

proportionate consumption of the available service in the

productive plant is generally accepted. The Depreciation

Committee of the National Association of Railroad and Utility

Commissioners pointed this out as follows:

A proportionate part of the cost of the
sewing machine is as much a part of the cost of
the pair of trousers as is the cloth of which
they are made or the labor expended thereon.
The economic life of the blast furnace is con-
sumed in turning out iron; the locomotive, ton-
and passenger miles; the generator, kilowatt
hours; the gas retort, cubic feet of gas.
(32, p. 54)

This allocation of plant investment to production costs is cost-depreciation.

The present study has been designed with cost-depreciation concept in mind. Therefore, all subsequent reference will be to this particular interpretation unless otherwise indicated.

Depreciation Accounting

Current depreciation accounting

The objective of current depreciation accountancy is to produce a realistic statement of production costs experienced over the accounting period. Since these costs include some provision representing the dissipation of the ability of the plant to produce, current depreciation accountancy and cost-depreciation are integrally related.

Under this system an estimate of the annual cost-depreciation is needed for each accounting period. This sum is then charged to production expense and credited to the depreciation reserve. If the annual charges have been estimated correctly, each year's production costs will be representative and the balance in the reserve will show the extent to which the service capacity of the surviving plant has been consumed as of any particular date.

It is apparent that the critical factor is the

determination of the annual depreciation accrual. Since
the basic measure of depreciation was defined as the consump-
tion of service, the number of units produced or the amount
of service rendered would provide the most logical basis
upon which the evaluation of the annual charge could be made.
Such an allocation procedure has been used and is known as the
unit of production or use method. It has had very limited
application, however, because of the difficulty in deter-
mining the proportionate service consumption, particularly
when a number of machines are used in varying amounts on a
number of products or services. Another difficulty of this
approach is that of making a reasonable estimate of the total
expected service in terms of production units. Despite
these handicaps, certain firms, e.g., the Canadian Pacific
Railroad, have had considerable success with this method of
allocating depreciation expense (47).

Most all industrial organizations prefer to express ex-
pected service in terms of service life. This convention is
normally referred to as age-life depreciation. Two general
approaches are available: (a) whole life in which the year's
cost-depreciation is related to the portion of the average
service life consumed that year, and (b) remaining life in
which the annual depreciation is determined by that part of
the average remaining life or expectancy consumed during the
year. The whole life concept is most commonly adopted but

the remaining life procedures have received considerable
attention from certain groups, e.g., the California Public
Utilities Commission (9) and others (10).

Depreciation base

Original Cost, replacement cost, and value have been
proposed as the correct sum to be recovered through the de-
preciation charges. Whichever is adopted is called the de-
preciation base.

Original cost. The original cost basis is most always
used for accounting purposes as it is the only one that is
fully compatible with the concept of cost-depreciation. All
depreciation determinations for income tax purposes must be
made on this basis except in the case certain special
property acquisitions (34, 44).

Value. Fair value has been frequently presented as the
most appropriate basis for the depreciation charge. One of
the most notable arguments in behalf of this approach was
given by the Special Committee of the American Society of
Civil Engineers in their review of the 1943 report of the
Committee on Depreciation of the National Association of
Railroad and Utility Commissioners. They summarized their
conclusions as follows:

> In the opinion of the society's committee it
> (the 1943 Report) fails to present adequately the
> fundamental conception that loss of value to

property through age and use is inescapable and
that the extent of that loss of value at any date
should be the basis of all accounting and finan-
cial policies with reference to depreciation.
(3, p. 890)

Dean T. R. Agg, who was a member of this reviewing committee,
and Dean Anson Marston used the value basis in their develop-
ment of the present worth theory of depreciation (29). Scharf
and Leerburger (38) have also presented numerous papers in
support of this approach.

The principal objection to the use of value as the de-
preciation base for accounting is that the annual charge would
not reflect that which was actually spent to produce the
service. The National Association of Railroad and Utility
Commissioners argued that:

    With fair value depreciation base, the re-
corded cost of utility service becomes a hybrid
quantity comprised of actual costs for labor,
materials, etc., and a portion of the value of
the property. The result is neither cost nor
value. (32, p. 54)

Replacement cost. Interest in the replacement cost
basis seems to increase whenever the nation's economy ex-
periences a definite trend of rising costs. One of the main
arguments in support of this approach is that original cost
depreciation charges are not enough to replace the old
equipment when it is retired. Rising costs make any accrued
funds inadequate. Another contention is that the low depre-
ciation charges result in overstated profits and, subsequently,
too high tax assessments.

The major objection to replacement cost is essentially the same as that expressed against the value basis. This is simply that the cost of production should reflect the actual expenses incurred. The depreciation charge is not made, fundamentally, to supply new plant but rather to allocate the investment in the present plant to operating expense.

In a recent publication Brown (8) estimated that, had replacement cost depreciation been used instead of original cost, depreciation charges would have been increased by about $2 billion in the early postwar years and $2½ billion in 1950 and 1951. This in turn would have caused a $1-$2 billion decrease in tax receipts "in recent years". It was believed that the switch in depreciation methods would have reduced profits after taxes in 1948 by 30 to 50 per cent in most industrial manufacturing groups. Prof. Brown concluded (p. 17):

> Our general conclusions are that historic-cost depreciation is more desirable than replacement-cost depreciation for tax purposes. In our view, tax equity should be based on differences in real income. Replacement-cost depreciation ignores these differences by providing a special exemption for certain tax payers (depreciable-asset owners as opposed to financial-asset owners). Our analysis of the effects of the two methods on the stability of economic activity points to a slight favoring of historic cost depreciation. From the long-view viewpoint, historic cost depreciation in a period of inflation is likely to result in a smaller amount of capital formation than would replacement cost depreciation. Here we must weigh future against present consumer needs and consider the implications of this method

of financing opposed to alternatives. Finally, the
problem of a satisfactory measure of replacement-
cost depreciation seems unresolved.

Item or group accounts

Separate property accounts may be kept for individual

units or composite properties such as a building or a large

piece of machinery. These are known as item accounts. More

frequently the records for similar or like units are

gathered together into a single account and handled on a

group basis. If a new account is opened for each year's in-

stallations, the property in the account constitutes a vin-

tage group. When similar or like units of all ages are

grouped together, the account is termed a continuous group

or "open-end" account. This last form is by far the most

common.

The principal difference between item and group depre-

ciation is based upon mortality dispersion. Actually, there

is no dispersion in the item account since the unit is 100

per cent surviving until its retirement drops the figure im-

mediately to zero. In a group account a mortality pattern

will probably develop in which some units will be retired

quite early and others will remain in service a much longer

time. Under the item method the annual depreciation charge

is based upon the expected probable life of the property unit

so that the unit's cost will be recovered completely by the

date of retirement. Under the group method the annual charge
is based upon a representative average life which is a func-
tion of the mortality dispersion expected of the property.
The depreciation charges are continued in behalf of the
group until the last unit is retired.

Grant and Norton (20) contend that an overwhelming majority
of group accounts is due to Internal Revenue Service policy.
They note that prior to 1934, provision was made for a com-
pany to fully recover the original cost of unit which was re-
tired before being fully depreciated. However, on April 4,
1934 the Treasury Department issued Mimeograph 4170 which
said in part:

> Where an account contains more than one item
> it will be presumed that the rate of depreciation
> is based upon the average life of such units.
> Losses claimed on normal retirement of units in
> such an account are not allowable, inasmuch as the
> use of an average rate contemplates the normal re-
> tirement of assets both before and after the
> average life has been reached and there is, there-
> fore, no possibility of ascertaining any actual
> loss under such circumstances until assets con-
> tained in the account have been retired. (45)

This policy is amplified further by the following statement
from the 1942 edition of the United States Treasury Depart-
ment's Bulletin "F":

> Accounting losses from the normal retirement
> of assets are not allowable under any method of
> depreciation accounting unless, in the case of
> classified or group accounting, the depreciation
> rate is based on the expected life of the
> longest-lived asset in the group, and in item
> accounting only when the maximum expected life of

the asset is used, since correct item accounting
requires an accurate determination of the life
of each individual asset, which is a practical
impossibility until near the end of its life.
(44, p. 7-8)

These requirements obviously make anything but group accounts

most impractical in all but a few instances.

Unit and dollar designation

Complete property accounts will generally indicate the

number of units in service at any time as well as an exten-

sion showing the original cost investment in those units.

Estimates of service life and mortality behavior can be

determined from either of these designations. However, the

dollar basis is most always used for accounting purposes.

This is because the unit is rather difficult to define in

most group accounts. For example, an account for centrifugal

gas pumps will not only carry all such pumps but also may

include such accessory items as foundations and baseplates,

lubricating systems, power transmissions (shafting, pulley,

couplings, etc.), platforms, ladders, stairs and railings

(when an integral part of the pump). Under these conditions

the dollar basis is the only practical way to summarize the

amount of property involved.

Howard (21) has shown that dollar and unit designations

may produce varying mortality estimates if the property has

experienced price changes over the placement period. The

analysis of retirement experienced based on the number of
units in service is unaffected by price fluctuations. How-
ever, if the data are expressed in terms of dollars, the
behavior of the newer equipment is weighted differently from
that of the older units which might have had lower or higher
installation costs. While the variance in the estimates
appeared to be fairly small, no attempt was made to determine
its significance by statistical or other measures.

<u>Allocation techniques</u>

Ideally, cost-depreciation should be accomplished
according to the consumption of a plant's capacity to produce.
However, it is extremely difficult to get a valid measure of
the expiration of service capacity. Consequently, the
accountant assumes the annual decrease follows one of three
patterns. They are, first, a straight line, second, a curve
indicating decreasing annual increments, and, third, a curve
showing increasing annual increments. These assumptions were
all originally conceived for item depreciation but they have
been applied to group accounts with fairly satisfactory re-
sults. However, the graphical interpretation of a straight
line or a particular curve are not appropriate when the
methods are applied to continuous or "open-end" accounts be-
cause the additions and retirements change the depreciation
base, and, hence, the relative size of the successive annual

charges.

Straight line. The average life procedure for the straight line assumption is by far the most common method in use today. It is equally applicable to item or group accounts. The depreciation rate is a constant for any given measure of service life and salvage value and may be defined as

$$\text{Straight line rate} = \frac{1.00 - \dfrac{\text{Estimated salvage value}}{\text{Depreciation base}}}{\text{Probable life or average life}}$$

where the probable life is used for item accounting and the average life for the group computation. The concept of a straight line allocation suggests equal annual accruals. This is the case for the depreciation of a single unit since the charge, d, for any year, x, is

$$d_x = (\text{Item depreciation rate})(\text{Depreciation base}).$$

For group properties the expression for the annual accrual at any age, x, becomes

$$d_x = (\text{Group depreciation rate})(\text{Average fixed asset balance, year } x).$$

The average fixed asset balance is assumed to be one-half the sum of the account's beginning and ending balances for the year. This calculation appropriately allows a half-year's charge for those units retired or added to the property during the year. It is to be noted that the variable nature of a continuing asset balance prevents the equal annual accruals

normally expected of a straight line method.

Since the group rate given above is a function of the expected average service life, it is obvious that those units retiring before average life will not be fully depreciated when they are removed from service. Likewise, those remaining longer than average life will be over-depreciated. However, if the estimate of average life is correct, the total original cost of the group will be fully recovered as the last unit is retired.

Some authorities have disapproved of this delayed re-coupment. It is the feelings of these men that each unit of the property should be fully depreciated at its retirement (29, 39, 49). The allocation procedure which would theo-retically assure this recovery is termed the unit summation approach. To compute the annual depreciation expense by the straight line unit summation method, the engineer must pre-dict the complete mortality dispersion of the property. This is necessary since the units at any specified age within the property group will be expected to have varying lives de-pendent upon the dispersion. Likewise, each length of life will have a different straight line rate. Hence, the appropriate depreciation rate for any age is a weighted average of all the individual straight line rates necessary within the group.

The complexities of the rate determination considerably

limit the use of this procedure. Winfrey (49, 50) has ex-
plained the method thoroughly and Fitch (19) has presented
the theoretical development of both the average life and the
unit summation straight line methods.

Decreasing annual charge. The principal allocation
technique in this category is called declining balance.
Mathematically, the annual accrual for an item of property at
age x is defined as

$$d_x = (\text{Declining balance rate})(\text{Undepreciated book balance at beginning of the year } x)$$

where the undepreciated balance is the depreciation base less
the balance in the depreciation reserve at the beginning of
year x. The declining balance rate has been frequently ex-
pressed in terms of the depreciation base and expected sal-
vage (20, 30). That is,

$$f = 1 - \sqrt[n]{\frac{V_s}{B}}$$

where f = declining balance rate

  n = probable or average life

  V_s = estimated salvage value at age n

  B = depreciation base.

This expression is normally of little consequence, however,
since the rate, f, is limited to twice that of the straight
line rate when the depreciation is computed for income tax
purposes.

The annual charge for a group property is found in

exactly the same way as the item accrual if there are no
additions made to the account during the year. In case new
units are added, a half year's charge is made in their behalf,
or,

$$d_{additions} = (\tfrac{1}{2})(\text{Declining balance rate})(\text{Depreciation base of year's additions})$$

This sum is added to the annual expense computed for the
units in service as of the beginning of the year to get the
account's total accrual.

The constant rate applied to the diminishing undepreciated
balance produces decreasing annual depreciation allocations
for item or vintage group properties. The early years, how-
ever, have considerably greater accruals than the amounts
based upon the corresponding straight line rates. Thus,
faster recoupment of the plant investment is possible by this
method, at least, in the case of newer properties.

The sum-of-the-digits method also gives heavy, early
accruals. Under this plan any year's annual charge for an
item account is determined by

d_x = (Sum-of-the-digits-rate for year x)(Depreciation base).
Mathematically, the rate is described by

$$d_x = \frac{2(n-x)}{(n+1)(n)}$$

where n = probable life

x = age of unit (limited to integral values) at be-
ginning of year for which the charge is desired.

When fractional years are involved the appropriate percentage of the full year's allocation is made. This procedure is limited to item accounts although proposals for handling group properties have been made to the Internal Revenue Service (36). Grant and Norton (20) have presented a complete discussion of the applications and uses of the faster write off procedures.

Increasing annual charge. The so-called interest methods, sinking fund and present worth, produce increasing annual accruals with age. As such, they tend to approximate the actual decline in physical condition of some properties.

The sinking fund approach was originally discussed by E. A. Sailers in 1915 (37). It has been used to limited extent, principally in the public utility industry. However, a 1953 survey of the National Association of Railroad and Utility Commissioners showed less than 6 per cent of the class A and B privately owned electric utilities were still using this method (17). An example of sinking fund depreciation for both the item and group accounts was presented in the 1943 report of the Depreciation Committee of this Association (32).

Marston and Agg proposed a present worth approach in which the depreciation was measured in terms of the present worth of the probable future service expected from the property (29). Winfrey (49) has discussed the method as has

Slade (39). This is a value-depreciation plan and, hence, ought not be used for cost-depreciation.

Depreciation reserve

While the term, reserve, commonly implies an actual fund or surplus account, the depreciation reserve represents nothing but an estimate of accrued depreciation. The Accountant's Handbook states:

> The conventional depreciation reserve not only does not consist of an actual fund but can be said to represent or indicate a fund of property only in a very indirect way, if at all. The balance in the reserve or allowance for accrued depreciation represents just one thing - the estimated expiration in use of the depreciable property shown in other accounts at cost or other gross book value. (34, p. 747)

The reserve for depreciation is what is known as a contra or valuation account. That is, annual depreciation accruals are recorded and summed in the reserve so that the extent of depreciation in some other account may be known or valued at any time. Actually, such nominal accounts as the depreciation reserve are merely an accounting convention since the annual depreciation charge could be credited directly to the fixed asset account without affecting the over-all balance. Many accountants essentially do this assignment when they carry a net fixed asset figure on a company's balance sheet. The net amount represents the difference between the original cost of the plant and the depreciation

reserve. This practice is fairly widespread in manufacturing industries but is quite rigorously opposed by public utilities executives who are concerned about possible misinterpretation of the resulting financial statements by those parties not fully cognizant of accounting procedures (1, 16).

Thus, while the reserve account does indicate the extent of depreciation accomplished, it does not represent an amount of dollars held or "reserved" for replacement or a "rainy day". The funds withheld from earnings for depreciation may be kept in the firm's general funds, or assigned to an actual cash savings fund, or spent for new equipment, etc. But the reserve balance does not show which, if any, of these have occurred. The nature of the depreciation reserve may be further emphasized by noting that the full amount of an annual depreciation charge would be added to the reserve balance even though no actual earnings were realized by the company for the particular year.

Finally, the reserve represents accrued depreciation for those units in service only. Upon retirement, the original cost or other basis of the unit is subtracted out of both the fixed asset account and the reserve. Under group accounting there is no attempt to assign a portion of the reserve to each unit. Instead, the balance represents consumed service from the survivors of the account as a whole.

Adequacy of reserve. If effective financial control is

to be maintained, it is highly desirable to frequently determine how reasonable or realistic a firm's depreciation policy has been. Of perhaps more importance is the consideration of what the future policies should be. An evaluation of present condition is usually a preliminary to both these investigations. The depreciation reserve represents the present status and, hence, the adequacy of its balance provides the basis for the policy appraisal.

The Depreciation Committee of the National Association of Railroad and Utility Commissioners has presented examples illustrating the two general approaches for determining the adequacy of the depreciation reserve. These are termed retroactive and prospective (32). The former procedure involves going back to age zero and building up a reserve with the appropriate annual charges less the deductions for retirements. The latter approach indicates the proper reserve at a given age by considering future expected accruals and future expected retirement debits.

In general, the prospective method is preferred because of simpler calculations and the applicability of formulas. For example, the reserve requirement ratio for units of a certain age x as indicated by the prospective straight line average life approach is

Reserve requirement ratio, age x =

$$(1 - \frac{\text{Estimated salvage at retirement}}{\text{Depreciation base}})(1 - \frac{E_x}{E})$$

where E_x = average expectancy of units at age x

E = average life of all units in group.

This ratio is applied to the original cost of the units of age x. The resulting product is the number of dollars which should currently be credited to the reserve in behalf of this age group. The calculated total reserve balance for the particular account would be the sum of the requirements of all the age groups represented. Likewise, the summation of the various account balances would indicate the necessary reserve requirement for the company as a whole.

It is apparent that the reserve requirement is a mathematical computation which must be considered in light of the assumption necessary to permit its determination. Logan (28, p. 163) has indicated that these are as follows:

1. That the property, for which the computation is made, will experience a specified average life span;
2. That the mortality or dispersion pattern is known, i.e., what percentage of all units installed in a given year will survive at each age;
3. That the presently surviving units of property, for which the reserve is computed, have precisely conformed to assumptions 1 and 2; and
4. That the future amounts of net salvage (or removal cost) of the presently surviving units are now known.

Thus, it is seen that a prime requisite in the evaluation of

the adequacy of the reserve is a carefully made prediction of the expected mortality dispersion.

Reserve adjustments. While the reserve computation is at best an approximation or an estimate of what the reserve ought to be, the actual balance is sometimes found to be so far from the expected value that some adjustment is felt necessary. Two alternatives for making the corrections are available. One is to accomplish an immediate transfer to or from the company's surplus to the depreciation reserve. The other is a gradual elimination of the discrepancy by increasing or decreasing the depreciation rate slightly. Both methods have their undesirable features, unfortunately. If an appropriation is made from surplus to reserve, or vice versa, the new balance may be reasonable, but the depreciation indicated still has not been accrued. Production costs in the past were not properly adjusted for depreciation expense. Likewise, the modification of the depreciation rate tends to distort the future income statements.

It appears that the wisest policy is one of frequent analysis to determine the reserve requirements and the expected mortality dispersion. If depreciation rates are kept current as a result of these studies, the reserve will not need adjustment.

REVIEW OF LITERATURE

The topics of mortality analysis and of depreciation have received considerable attention in the literature. Many of the more widely recognized publications have been reviewed or noted in the preceding sections. It is felt that further elaboration would merely duplicate the work of previous authors, e.g., Fitch (19), Saliers (37), Winfrey (49). Consequently, the following discussion is concerned principally with the specific studies and reports which suggested the present investigation.

In 1935 the Iowa Engineering Experiment Station issued its Bulletin 125, Statistical Analyses of Industrial Property Retirements (48). In this publication Winfrey reported the results of a curve fitting experiment conducted with the retirement experience he had used in the development of the 18 Iowa type curves. Each of the original sets of data was partially replotted to produce a stub terminating at 90 per cent surviving. The resulting curves were smoothed and extrapolated by one of the Iowa types so that an estimate of average life was obtained. Another indication of service life was determined after each of the stubs had been extended to 80 per cent surviving. The process was repeated at 70 per cent surviving and so on.

As might be expected the variation in the average lives estimated from the fittings decreased considerably as the length of the stubs were increased. For example, the average error of the predications was found to be roughly ± 20 per cent of the correct value for data ending at 90 per cent surviving and ± 3 per cent at 0 per cent surviving.

The deviation in the estimates for complete survivor curves is rather interesting since the standard types were based upon these same data. Winfrey explains this variation by noting that many of the curves fell between two types and could be classified as either one of them. He also mentioned that some difficulty was encountered in fitting data having high modal frequency curves. The survivor curves in these instances would be quite steep and, thus, hard to classify as to type except for the trends indicated at the ends. It was suggested that a comparison on the basis of frequency curves would be more successful for data of this type.

The 1943 report of the Committee on Depreciation of the National Association of Railroad and Utility Commissioners presented an extensive review of most all phases of depreciation and depreciation accounting (32). This particular work was not accepted formally by the Association at its meeting that year but was merely circulated among the member commissions. The major objections to the report were not voiced against the manner in which the various techniques were

described or evaluated. Rather, the principal point in contention was the recommendation made by the Committee that all depreciation calculations be based upon the straight line approach under a current depreciation accounting system. Most utilities at that time were using a somewhat different procedure known as retirement reserve accounting. Quite naturally these firms were concerned about the problems of a basic shift in accounting procedures (1). It is interesting to note, however, that by 1953 over 90 per cent of the privately owned class A and B utilities had adopted the recommended system (17).

Using an experience band from a hypothetical property experience, this Committee illustrated the use of the Gompertz-Makeham equation, the Iowa type curves, and the fitting of retirement ratios by orthogonal polynomials. Two significant comments were made concerning the Gompertz-Makeham approach. The first, and most important, was the factor in the equation which was to allow for retirement from "fortuitous causes" becomes negative in the analysis of property behavior. Since most industrial equipment is retired for reasons which may be classified as fortuitous rather than related to age, the report questioned the desirability of using this method of analysis.

The second observation noted was that frequently the life table resulting from Gompertz-Makeham equation reaches a

maximum before age zero and, at times, this value may exceed 100 per cent. However, the report indicated that if this occurs at a very early age, the data could be arbitrarily smoothed to eliminate the difficulty.

With regard to the fitting accomplished using the Iowa type curve, the Committee commented (p. 244):

> Where the stub life tables run to, or nearly to, the point where 50 per cent of Radix survives this method will ordinarily yield satisfactory estimates of average service life. However, the estimates of mortality dispersion leave something to be desired. In other words, while this method appears to yield fairly reasonable estimates for annual depreciation, it is likely to be less satisfactory for accrued depreciation.

No indication was given in the report whether these conclusions were based upon the results of this single study or whether further analyses were accomplished.

A second degree equation developed by orthogonal polynomials was used to smooth the retirement ratios from these same data. It was reported that a comparison of the resulting life table with the actual data was quite good. The absence of any specific criticism such as accompanied the discussions of the other methods of analysis appears to indicate that the Committee found this particular fitting technique to be superior.

The cooperating Committees on Depreciation of the American Gas Association and the Edison Electric Institute issued a survey of mortality analysis procedures in 1942 (2).

Their study made use of actual mortality experience from two accounts, "Electric Meters" and "Water Gas Sets". However, all comparisons between the various methods were based on analyses of complete life tables. The applications were therefore limited to those of smoothing data. Under these particular conditions, all actuarial procedures were in good agreement on estimates of average service life.

Orthogonal polynomials were not used in this study. Rather, the retirement ratios were fitted to third degree equations using conventional least squares methods. This permitted the use of data weighted with dollars in service at each age. It was reported that the weighted curves agree with the other actuarial methods better than the unweighted. This is particularly interesting because the least squares solutions by orthogonal polynomials uses unweighted data.

A complete expression of mortality dispersion is used principally in the determination of the adequacy of a depreciation reserve balance. Logan (28) investigated the problem of how much the calculated reserve would be changed by the assumption of various behavior patterns. He obtained dispersion estimates for a band of actual experience using the Iowa type curves and the simulated plant-record method. A very satisfactory fit was obtained from the standard curves but the plant-record approach did not clearly indicate whether the appropriate survivor curve was based on a left,

or right, or a symmetrically moded frequency distribution. Comparing the possible "acceptable" solutions, Logan found a 19.5 per cent difference between the maximum and minimum reserve figures based upon straight line depreciation calculations. He concluded that the range of "correct" balances indicated that it was unrealistic to force a reserve to conform to the results of a single mathematical calculation. Instead, he proposed that any adjustments be deferred until a series of reserve requirements evaluated over a period of years could be studied and analyzed for trends.

PROCEDURE

One of the principal purposes of the study was to pro-
vide useful information to the practicing depreciation
engineer. Consequently, every effort was made to conduct the
experiment under conditions that closely resembled those of
an actual analysis of retirement experience.

The general outline of the investigation was to submit
the same incomplete or stub data to analysis by both the Iowa
type curves and orthogonal polynomials and to then compare
the corresponding results. A panel was deemed advisable in
the case of the type curves since considerable judgment is
exercised in matching the data. One person might be subject
to a particular bias and, therefore, not fairly represent the
system. On the other hand, any significant bias noted in an
analysis of a panel's results would likely be characteristic
of the method, and, as such, would be extremely important.
Only one calculation was accomplished for the orthogonal
analyses since the approach is mathematical and, under a
given set of conditions, will always produce the same results.

The Data

The principal criterion used in the selection of the original property experience was that a retirement rate analysis of an appropriate observation band would produce a complete or nearly complete life table. It was desired to have enough experience in each account to cause the per cent surviving values to extend down to the range of at least 10-15 per cent. The specification of the retirement rate analysis was made because this approach is used almost exclusively in studies of total account behavior. Complete life tables were required since they would provide the standards for the comparisons. That is, the function of the fitting technique is to recognize the trend or mortality "law" present in the incomplete experience and then to extrapolate the data on that basis. Therefore, all the test stubs were merely the higher per cent surviving portions of complete or nearly complete life tables. This procedure allowed the predicted results to be compared with the complete, actual life table.

Roughly 75 curves which met the above requirements were collected from various firms, principally in the gas and electric utility industry. As each set of data was received, it was assigned a code number from a table of random numbers. No particular attention was paid initially to the classes of

property represented by the data. Rather, the emphasis was upon getting a range of types of mortality dispersion so that the tests would be fairly representative of the situations encountered in practice. Therefore, the data were plotted as survivor curves and roughly classified according to the general Iowa types. This was done to see how well the study would cover the possible variations in mortality dispersion as given by these widely accepted standards of industrial property retirement. It was found that 15 of the 18 Iowa types were represented. Specifically, versions of the L_0, L_1, L_2, L_3, L_4, R_1, R_2, R_3, R_4, R_5, S_0, S_1, S_2, S_3, and S_4 curves had been received.

Next, two samples from each type available were selected so that a long and a short average service life experience was specified for each general mortality classification. In the cases of the S_2, S_4, and R_5 types only one sample of each was found. For these curves the second set of data was synthesized from the first by plotting only a part of the points to a correspondingly reduced scale, e.g., every other point was plotted with the scale of the age axis reduced by a factor of two.

A few of the test curves chosen did not have complete life tables. The type classifications of these data were re-checked carefully and then used as a starting point in extrapolating the curves to completion. Every effort was

made to continue the data according to the trends established.
In one or two instances the type curve which was most
appropriate for the over-all experience did not give a
smooth extension at the advanced ages and in each case it was
discarded in favor of a more logical development. One curve,
No. 27, had departed so significantly from any Iowa type by
the time it had reached 10 per cent surviving that the
extrapolation was done by eye.

Only those life tables which did not go to zero were ex-
tended or smoothed in any way. If an extension was necessary,
the original data were not touched until an age in which the
dollars remaining in service represented less than 1 per
cent of the maximum in service balance in the experience
band. A single exception to this was life table No. 21 in
which the total dollars in service at the age the extrapola-
tion began was slightly under 4 per cent of the maximum
figure. Since the experience for the advanced ages and the
small balances is normally considered somewhat questionable,
it is believed that these extensions had little, if any,
effect on the over-all results.

A brief summary of the classes of property used in the
study was prepared as an aid to the panel in their fitting
procedures and has been included in Appendix B, p. 128 . All
data were classified according to the account numbers as
specified by the United States Federal Power Commission in

its uniform system of accounts for natural gas companies (41) or public utilities (42). The descriptions for the three life tables synthesized from other experience are, of course, assumed. These particular tables were Nos. 20, 26, and 39. The properties described have been known to behave similarly to the curves represented, however.

To obtain the stub data for the study, each of the 30 life tables was arbitrarily terminated at approximately 70 per cent surviving. The resulting group of incomplete data were termed "heavily stubbed" and the appropriate curve numbers modified by the code letter C, e.g., No. 10-C. Another group of 30 stub curves was developed by again taking the data from the original experience and terminating it in the vicinity of 50 per cent surviving. These curves were referred to as "lightly stubbed" and their numbers were modified by the code letter S, e.g., No. 10-S. The word designations, heavily and lightly stubbed, were preferred since the erratic nature of the data prevented the ending of the test life tables at exactly 70 or 50 per cent surviving. Actually, while these values were approximated in most cases, the lightly stubbed terminal points ranged from 30 to 60 per cent surviving and the heavily stubbed values extended from 60 to 90 per cent.

Tables of retirement ratios were prepared so as to include exactly the same amount of experience as was given on

the survivor curves. This required that the last ratio included in any data set be the one which expressed the expected retirement for the year preceding the terminal point of the corresponding stub curve.

While the development of the test curves was accomplished using the survivor curve representation of the original data, a plot of the observed retirement ratios could have been used as well. However, the former approach was preferred since the extent of retirement experience is normally expressed in terms of per cent surviving.

During the stubbing process no consideration was given to the resulting shape of the curve or whether the included points seemed to indicate the general trend of the complete curve. Attention was given only to the selection of terminal points which approximated the desired values and to the establishment of an adequate differential between the data included in the heavily and lightly stubbed curves.

The Iowa Type Curves

The panel

The panel consisted of 16 members, all of whom had had some experience with the Iowa curves. Panel members were selected mainly upon the recommendations or references of firms or personal acquaintances who were familiar with their

work. The names of the panel members, their business addresses and positions, where known, are given in Appendix A of this dissertation. Vocationally, the membership may be summarized as follows:

4 -- consulting depreciation engineers

1 -- valuation engineer

4 -- college professors or teachers

2 -- United States government service engineers

2 -- public utility commission engineers

3 -- public utility depreciation engineers.

The actual experience with the use of the Iowa curves varied considerably within the panel. In general, those in teaching were not actively engaged in depreciation work but had taught the use of the curves for some time and used them themselves to a limited extent in the analysis of industrial retirement data. Three of these four members had served at one time on the faculty of the Iowa State College. However, only one is so engaged at the present time. The experience summary of the panel, excluding teaching personnel, is given in Table 1.

Analysis of the data

In order to make a uniform presentation of the data to the panel members, the incomplete life tables were reproduced as stub survivor curves on tracing paper. The only

Table 1. Experience summary of panel members[a]

Experience in using Iowa type curves, years	Number of panel members
0 - 1	2
1 - 5	2
5 - 10	2
10 - 15	2
Over 15	4

[a]Summary does not include the four members whose experience was principally based upon teaching the use of the system.

information appearing on these plots was the data points, the curve number and code letter, and the appropriate scale markings. A sample of one of these reproductions has been presented.[a]

To assign the curves for analysis, the 60 stub curves were first divided into two groups of 30 according to the long and short average lives. Next, the 15 heavily stubbed curves were selected from each group and exchanged. Thus, both curve series consisted of 30 stubs, half of which were derived from the longer-lived examples, and half were prepared from the shorter-lived versions. Likewise, each of these sets included 15 heavily stubbed data and 15 lightly stubbed

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[a]_Infra_ Appendix B.

curves. However, one group had the heavily stubbed, long life curves and the other had the lightly stubbed, long life curves. Similarly, the first set had all the lightly stubbed, short life data, and the second had the heavily stubbed, short life curves.

At random eight panel members were assigned the first series of curves and the remaining eight men received the second series. Each member was given a series of 30 stub curves, a set of instructions, a brief description of the properties included in each account represented in the curves, a summary sheet upon which to record his fittings, and two sets of Iowa type curves. One of these sets was a series of the standard curves which had been prepared by the Iowa Engineering Experiment Station. Each mortality type had curves plotted at five-year intervals in average life.[a] The other set presented each type curve in one-year average life intervals.

No fitting instructions were specified because it was felt that the analyst ought to use the particular routine with which he was familiar. However, comments from the panel indicated that the most widely followed procedure was one of first drawing in a curve through the data by eye. With this freehand curve as a guide, the engineer determined the

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[a]Infra, Appendix B.

appropriate type and the approximate average life by matching the data to the five-year interval curves. This fitting was then checked and verified using the one-year curves.

One departure from the practical curve fitting situation was suggested in the set of instructions. The panel member was asked to assume that all data given were developed from essentially the same or at least an adequate amount of experience. The purpose of this qualification was to eliminate any question as to the appropriate weight to give to the various points, particularly those at the more advanced ages. This assumption was not unreasonable in this instance since all the data shown were actually the upper portions of complete life tables. However, in fitting a stub, the analyst would normally have to satisfy himself as to the validity of the data appearing in the terminal portion of the curve.

Orthogonal Polynomials

This method of curve fitting was applied directly to the observed retirement ratios which were chosen in preference to survival ratios because the former were smaller and easier to handle mathematically. The fitting procedure, which embraces the concepts of least squares, resulted in a polynomial expression relating the ratios to age. Sample calculations illustrating the orthogonal approach are given in Appendix D.

In proposing this particular method, the engineers of
the New York Public Utilities Commission suggested that the
second degree polynomial was used most often but that
occasionally either the first or the third would be appro-
priate (33, p. 79). The problem of smoothing retirement data
is not one of merely getting the closest fit to the observed
data. The resulting equation must not produce inappropriate
ratios. That is, the supposition is made that as the property
grows older its probability of survival through the next age
interval decreases. This general assumption ought not be
discarded for a better fit over the earlier years. Hence,
first or second degree equations are normally preferred.

For the work included in this study, second degree ex-
pressions were used wherever possible. Two exceptions should
be noted. The first was in the case of a few sets of data
which indicated fairly rapid retirement in the early years
and then a pronounced diminishing of the rate, followed by an
increase again. Third degree solutions were preferred for
data of this form.

The second situation in which a particular degree was
not appropriate occurred because of a mathematical charac-
teristic involved. The retirement ratio may be defined by

$$R = \frac{-\Delta y}{y \, \Delta x}$$

where R = retirement ratio for a unit time interval

$\triangle y$ = retirements experienced over any time interval

y = survivors at the beginning of the time interval

$\triangle x$ = time interval.

Since the fitting process expresses the ratio as a polynomial which is a function of age x, the following may be written:

$$R = -\frac{1}{y}\frac{\triangle y}{\triangle x} = a + bx + cx^2 + \ldots$$

where x = age

a, b, c, etc., = constants determined by the data.

This equation approximates

$$\frac{1}{y}\frac{dy}{dx} = -(a + bx + cx^2 + \ldots)$$

which upon integration becomes

$$lny = K - ax - \frac{bx^2}{2}\frac{cx^3}{3}\ldots$$

or

$$y = e^{K - ax - \frac{bx^2}{2}\frac{cx^3}{3}}\ldots$$

The resulting equation is recognized as the mathematical expression for the life table or the survivor curve. However, in a series such as the one present in the exponent of e above the leading term, i.e., the term with the highest power, predominates as the variable increases in magnitude. Thus, if the life table values, or y, are to go to 0 per cent, the leading term of the exponent must be negative. Fortunately, it is possible to determine the sign of the leading term before it is actually evaluated when data are fitted with orthogonal polynomials. This feature permits the elimination

of those degrees which are inappropriate and aids the selec-
tion of the best equation form.

With regard to the actual calculations involved in the
fitting process, three assumptions were made to provide a
consistent and reasonable approach to each set of data. The
most significant dealt with weighting the ratios. In the
original presentation of the method the suggestion was made
that at advanced ages the data might become erratic and
probably should be rejected. The explanation for this was
that as the analysis reaches the extreme ages, the amount of
data available decreases markedly and it is doubtful that the
results could be considered representative. Thus, it was
left to the engineer to run a number of analyses, each time
excluding more or less data, until a satisfactory fit was ob-
tained. This is a perfectly reasonable approach for a par-
ticular study but it could not be followed in the present in-
vestigation because, in order to be valid, a comparison must
be made under the same conditions, i.e., using exactly the
same retirement experience. Therefore, the assumption was
made that all ratios given resulted from essentially equal or
at least adequate experience. Consequently, all data given
in the stubs were used in the fitting process regardless of
the trend exhibited at the older ages. It will be recalled
that this same condition was applied to the Iowa type curve

fitting procedure as well.[a]

The second assumption dealt with the question of the proper interpretation of negative retirement ratios. At times the mathematical expression would produce values for the smoothed ratios which became negative before age zero. While these were perfectly correct from the mathematical point of view, they were inappropriate for describing re- tirement experience since they indicated an increase in the units surviving. One of two alternatives was always selected. If the values became just slightly negative and then positive once more, the ratios for those particular years having negative terms were assumed to be zero. This was particularly done in those cases where the fit of the data was improved by such an assumption. On the other hand, if the smoothed ratios remained negative for a number of age intervals, the ratios were assumed to be zero for all intervals preceding the year in which the values became and remained positive.

Finally, the problem of computing the appropriate ratio for the first half year presented some difficulty. The use of the orthogonal solution requires that the independent variable, age, be increased in equal increments, i.e., one year intervals for ages $\frac{1}{2}$ to $1\frac{1}{2}$, $1\frac{1}{2}$ to $2\frac{1}{2}$, and so on. There- fore, there was no way to evaluate directly the rate for age

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[a] Supra, p. 49.

0 to $\frac{1}{2}$ unless the regression equation was determined. This expression was not needed for any of the other values so it was quite inconvenient to make the evaluation for this single term. Therefore, the ratios for this first interval were defined by adopting one of two arbitrary solutions. When the succeeding ratios were zero, the first ratio was assumed to be zero, also. In any other instance, the ratio for the age interval $-\frac{1}{2}$ to $+\frac{1}{2}$ was found. If it proved to be positive, one-half of its value was assumed to be the correct ratio for the 0-$\frac{1}{2}$ age interval. If the calculated value was negative, the ratio for the first interval was assumed to be zero.

TREATMENT OF RESULTS

The investigation was designed to permit an evaluation of the results on the basis of statistical analyses. However, any interpretation of the significant tendencies indicated is dependent upon the standard adopted, the measures of fit compared, and the analysis procedures used. Descriptions of these factors as they pertain to the present study are presented in this chapter.

The Standard

The purpose of the curve fitting method is to "recognize" the trend of the data exhibited in the stub curve and then to extrapolate the experience on this same basis. For this reason all stubs used in this study were developed from complete survivor curves. This permitted the adoption of the full life table[a] as the standard against which the various predictions were compared. No smoothing of these was done because this would have forced some particular form upon the experience. Rather, it was felt that the original pattern,

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[a]Supra, p. 56. The full life table may include some smoothed data added for the advanced ages in order to complete the survivor curve.

no matter how erratic or illogical, provided the fairest
standard for the comparison.

Comparison Bases

Numerous measures designed to test for closeness or
goodness of fit were considered. For example, the vertical
distance between a standard or correct curve and the predicted
dispersion was noted. However, in some instances the Iowa
type curves chosen by a few panel members terminated com-
pletely before the ordinates of the standard curve reached 25
per cent surviving. The appropriateness of any "distance"
measure beyond the age at which a curve reached absolute zero
was questioned.

Similarly, the difference in the ages at which the
standard and the predicted curve reached a certain per cent
surviving was tried. This amounted to a consideration of the
horizontal distance between the curves. The difficulty re-
sulting from data which became absolute zero was eliminated
but the computational problem was increased somewhat. Since
the ordinates of the Iowa type curves are normally recorded
in life table form at integral intervals in age or per cent
of average life, the age at which a particular percentage of
survivals occurred had to be obtained by interpolation for
each prediction of mortality dispersion.

The total area included between a plot of the standard
data and that of the estimated curve also provided a measure
of the difference in the two dispersions. All segments of
area were considered to be positive in sign, so that the full
variation was indicated. This gave a somewhat different com-
parison than that of the total areas under the curves, since
a portion of area excluded in one region was not balanced by
one included elsewhere.

Any mathematical evaluation of this measure using the
calculus was made impractical by the complex expressions for
many of the Iowa type curves. A satisfactory, but laborious,
alternative was to plot the standard and the estimated dis-
persions and determine the area difference by the use of a
planimeter.

Each of the suggested measures posed some computational
difficulties, but none of the obstacles were insurmountable.
However, the question of the interpretation of the results
remained. In each case it was felt that any specific trends
indicated would have to be re-evaluated in terms of either
accrued or annual depreciation before its full significance
would become apparent. This necessity suggested the use of
these two phases of depreciation as the original bases for
the comparison. Further investigation showed that the
measures could be computed conveniently from the data which
were available.

Annual accrual

It has been noted that the annual accrual under all depreciation allocation procedures currently in use is a function of the expected service life.[a] Consequently, the variation observed in the predictions of service life constitutes a practical measure of each analysis technique's influence upon the yearly depreciation charge.

The actual determination of average service life expected was relatively simple. When the Iowa curves were used, this information was estimated directly from the type curve overlay. The standard data and the orthogonal predictions were given in life table form so in these instances the average life was found by numberical integration,[b] or

$$\text{Average service life} = \frac{\text{Sum of life table terms} - (75\% + \frac{1}{4}L_{\frac{1}{2}})}{100\%}$$

where $L_{\frac{1}{2}}$ was the per cent surviving at age $\frac{1}{2}$.

Accrued depreciation

The balance in the depreciation reserve is an estimate of the depreciated portion of the investment originally made for equipment and facilities presently in service. Further, its size is a reflection of previous service life estimates,

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[a]Supra, p. 37-43.

[b]Supra, p. 10.

of the actual mortality dispersion experienced, and of the allocation procedure used.

Any review of depreciation policies normally includes a study to determine how well the present reserve balance agrees with the sum that is deemed necessary in light of present and expected retirement experience. The "correct" or adequate reserve balance is dependent upon the predicted mortality dispersion as well as the depreciation allocation procedure adopted. Thus, even for a given retirement pattern, a wide range of computed reserve balances is possible, depending upon the methods used to compute the annual charge.

Therefore, accrued depreciation does not provide a comparison basis as completely acceptable as does the annual accrual. In view of the widespread adoption of straight line accounting procedures, however, it was believed that reserve requirements based upon that particular method of allocation would provide a satisfactory measure.

If the straight line concepts are assumed, the appropriate reserve balance for any age group would be given as follows (34):

Reserve requirement, $, for property at age x =

$$\text{Sum of retirements during future years} - d \begin{bmatrix} \text{Sum of future average annual balances} \end{bmatrix} - \text{Future salvange}$$

where d is the constant straight line depreciation rate, or

$$d = \frac{1.00 - \dfrac{\text{Future salvage}}{\text{Original cost}}}{\text{Average life}}$$

$$= \frac{1.00 - s}{E} \, .$$

By letting the original installation cost of the property at age x be represented by P, one can write the reserve requirement expression as

$$(L_x)(P) - (d)(P)(\tfrac{1}{2}L_x + L_{x-1} + L_{x-2} + \ldots) - (s)(P)(L_x)$$

or,

Reserve requirement, age x =

$$P \left\{ \left[(1-s)L_x \right] - \left[\frac{1-s}{E} \right] \left[\tfrac{1}{2}L_x + L_{x-1} + L_{x-2} + \ldots \right] \right\}$$

$$= P \left[1-s \right] \left[1 - \left(\frac{1}{E}\right) \left(\frac{\tfrac{1}{2}L_x + L_{x-1} + L_{x-2} + \ldots}{L_x} \right) \right]$$

where L_x is the life table value at age x. Symbolically, this expression may be further simplified by recognizing that the summation of the life table values divided by the per cent surviving at age x is the expectancy, E_x, at that age. Thus, the calculated depreciated balance for the property at any age may be found by multiplying the original installation cost X of the property by the factor

$$\left[(1 - s) \left(1 - \frac{E_x}{E} \right) \right].$$

The total reserve requirement for the entire account is subsequently determined by summing the individual balances for each age group.

For the purposes of the comparison in this investigation,
the expectancy-average service life ratio, E_x/E, was the only
element of the above factor considered since it alone was
affected by a prediction of mortality dispersion. The ex-
pected salvage, as well as the original installation cost, was
a constant for any given account.

Further, it was felt that a complete summation of
expectancy-average life ratios for all ages was not needed.
Instead, a representative sample of three ratios was deter-
mined for each dispersion estimate. The ages at which the
ratios were computed were arbitrarily chosen by dividing the
total age range given in the stub data into three intervals
of equal length. An expectancy-average service life ratio
was found at the terminal age of each interval. These values
were then summed to give the comparison modulus for each
estimate of dispersion.[a]

The determination of the factors for the Iowa type
fittings was accomplished by referring to published tables
(15) which gave the expectancy at each age for every Iowa
curve. In the cases of the standard and the orthogonal
estimates the expectancies were found by summation of the
corresponding life table entries.[b]

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[a]Infra, Appendix D, p. 147.

[b]Infra, Appendix D.

Analysis Procedures

A review of previously reported work indicated that the
principal sources of variation in dispersion estimates were
reported as (a) the general mortality pattern encountered,
(b) the amount of experience available, i.e., the completeness
of the life table, and (c) the length of the average service
life. The analysis of the results from the present investi-
gation as the original selection of the test curves for this
study was accomplished with these factors in mind.

Classification of experimental results for analysis

The set of 30 test life tables was comprised of data
representing 15 of the Iowa type curves. Two versions of
each type were included. This permitted the grouping of the
comparison moduli which resulted from the fitting of like
mortality dispersion. Similarly, the fact that each life
table was fitted when heavily stubbed as well as when lightly
stubbed suggested the further classification of the results
according to the amount of data given or the extent of the
stubbing. The final consideration in the arrangement of the
data for analysis was that one of the two curves included for
each type was a short average life experience and the other
was a long service life form.

Thus, 36 estimates of mortality dispersion were developed

under each of the 15 general mortality patterns. That is, eight Iowa type curve fittings and one orthogonal analysis were completed for the heavily stubbed version of the short service life experience. The same number of dispersion estimates were reported for the heavily stubbed, long-lived data. Likewise, a comparable number of fittings was accomplished for the lightly stubbed forms of these same data. The complete tabulation of the average service lives and the expectancy-average service life ratio summations resulting from the type curve fittings of the panel as well as the orthogonal analyses has been presented in Appendix C of this dissertation.

Test data

Since it was desired to base the comparison of the curve fitting methods upon the agreement attained between the predictions of dispersion and the actual mortality patterns, the estimated service lives and ratio summations were not subjected to analysis. Rather, the differences between these comparison moduli and the corresponding standards for service life and ratio summation provided the variation to be studied. Thus, the final step in the preparation of the data for analysis was the determination of these differences for every dispersion estimate made on each test life table. The development of the test data from the L_0 average service life

estimates has been illustrated in Table 2.

The analyses

The major factor considered in the evaluation of the experimental work was the variation in the predictions of average service life or of the expectancy-average service life ratios about the accepted standard values. Consequently, an analysis of variance proved to be the most appropriate statistical treatment of the results.

It will be recalled that for any general mortality type eight panel members analyzed the heavily stubbed, long life curve and the lightly stubbed, short life version. Likewise, the remaining eight men worked with the lightly stubbed, long life data and the heavily stubbed, short life curve. This arrangement provided a logical basis upon which the panel's responses could be grouped for the comparisons.

The test data, i.e., the differences between the estimated comparison moduli and the corresponding standards, were placed into three separate categories for analysis. A complete classification of these data for the L_0 type experience is shown in Table 2 and is explained below.

The first consideration was with regard to fittings of the heavily stubbed data. These responses are tabulated in the first column under Test Data, Long Life in Group I and Short Life in Group II. Similarly, the results from the

Table 2. Arrangement of panel's average life estimates for analysis (type L_0 data[a])

Panel group	Panel member	Life estimates years		Test data years		
		Heavily stubbed, long life	Lightly stubbed, short life	Heavily stubbed, long life	Lightly stubbed, short life	Stubbing effect, long life
I	1	34	22	1	-2	3
	2	33	22	0	-2	2
	3	34	22	1	-2	3
	4	35	22	2	-2	3
	5	34	22	1	-2	2
	6	33	22	0	-2	3
	7	36	22	3	-2	3
	8	27	22	-6	-2	-6
		Short life	Long life	Short life	Long life	Short life
II	9	21	31	-3	-2	-1
	10	16	31	-8	-2	-6
	11	16	31	-8	-2	-6
	12	17	32	-7	-1	-5
	13	16	32	-8	-1	-6
	14	15	30	-9	-3	-7
	15	16	33	-8	0	-6
	16	16	33	-8	0	-6

[a]Standards: long life, 33 years; short life, 24 years.

lightly stubbed fittings were summarized as in the second column under Test Data and analyzed. Finally, the data were arranged to consider the effect of stubbing upon the estimates. This step was accomplished by noting the differences in the estimates resulting from the fittings of the heavily and lightly stubbed versions of any curve. That is, the first entry under the heading, "Stubbing effect, long life",

was found by subtracting panel member No. 9's estimate of 31 years for the long-lived data from member No. 1's estimate of 34 years. Likewise, the 31-year estimate of panel member No. 10 was subtracted from No. 2's 33-year prediction to get the second entry in the column, and so on, the estimate from the lightly stubbed data always being subtracted from that of the heavily stubbed version. A similar set of subtractions gave the values in Group II for the short-lived experience.

The corresponding expectancy-average service life ratio summations resulting from these same estimates of dispersion were similarly classified. Thus, two separate analyses of variance were accomplished for each of the three classifications just described. The pertinent mean square values and the corresponding degrees of freedom determined in the analyses of the test data appear in Appendix E.

DISCUSSION OF RESULTS

The objectives of this investigation could be classified into two general categories. They would be, first, a comparison of the Iowa type curve and the orthogonal polynomial methods of fitting industrial retirement experience, and, second, an evaluation of some of the characteristics likely to be inherent in the Iowa type curve method. As such this classification provides the logical topic headings for the following discussion of experimental results.

Iowa Type Curves and Orthogonal Polynomials

Comparisons of the two methods were made with respect to the general mortality dispersion considered and to the amount of stubbing. Eight responses based upon Iowa type curve fittings were reported for each of the two heavily stubbed curves classified under every mortality type. Likewise, each of these sets of heavily stubbed data was fitted with orthogonal polynomials. Thus, the analysis of variance was performed upon the test data from 16 panel responses and from two orthogonal fittings. Similar data were obtained for the lightly stubbed curves. The results obtained from each of these analyses for the 15 general mortality patterns are

summarized in Table 3.

Tests for statistical significance were based upon the variance ratio, that is the ratio of the mean square values found from results reported for each curve fitting method. A 5 per cent level of significance was used with two degrees of freedom for the orthogonal mean square and 16 for the Iowa method. The value of the F distribution for these conditions was obtained from tables prepared by Pearson and Hartley (35).

An inspection of the significant findings reported for either basis of comparison reveals that no consistent superiority is enjoyed by either curve fitting approach. The number of better fits by the orthogonals for the L-type curves and the prevalence of better Iowa fits for the R-type curves suggest some advantage may have been held by a method for particular classes of dispersion types. This possibility is explored further below.

L-type

Four significantly better estimates of dispersion were indicated for the L-type curves. All occurred on the heavily stubbed version of the data and three of the four tests were measured by the expectancy-average service life ratio comparison base. This information indicates that with either heavy or light stubbing both methods produced comparable

Table 3. Indications of significantly better estimates of mortality dispersion developed with the Iowa type curve method or orthogonal polynomial analyses

General mortality type	Method giving significantly better estimate			
	Average service life		Expectancy-average service life summation	
	Heavily stubbed	Lightly stubbed	Heavily stubbed	Lightly stubbed
L_0	--	--	Orthogonal	--
L_1	--	--	--	--
L_2	--	--	Orthogonal	--
L_3	--	--	--	--
L_4	Orthogonal	--	Orthogonal	--
R_1	--	Iowa	--	Iowa
R_2	Iowa	--	Iowa	--
R_3	Iowa	Iowa	--	--
R_4	--	Orthogonal	--	--
R_5	--	Orthogonal	--	--
S_0	--	Iowa	--	Iowa
S_1	--	--	--	--
S_2	--	--	--	--
S_3	Orthogonal	--	--	--
S_4	--	--	--	--

estimates of total service, that is, the total area included under the survivor curve. However, because of the nature of the expectancy-average service life ratios, it appears that under certain conditions the orthogonal polynomial analysis may describe a mortality pattern which is in closer agreement with the standard data throughout the entire length of the

curve.

An examination of the specific experience fitted shows that the conditions under which the orthogonal approach proved to be superior were related to the marked departure of the retirement data from those of the left-moded Iowa types. For example, life table No. 27, the short life experience of the L_0 classification, seemed to cause the panel much more difficulty than No. 10, the other L_0 experience. The plot of the former life table has been described as departing considerably from any Iowa type by "tailing out" at advanced ages and approaching the axis almost asymptotically as do the so-called J- or O-type curves.[a] Since the standard for the comparison was computed on the basis of the complete table, it would be impossible for any choice of Iowa type to give very close agreement.

Further, the heavily stubbed version of No. 27 gave little or no indication of "tailing" nor did it show pronounced L_0 characteristics. This latter factor becomes particularly apparent by noting only one of the eight panel members who fitted this stub selected an L_0 classification. Six men chose S_0 and one R_1 and all underestimated the average life. However, for the longer or lightly stubbed curve the test data was extended on from 70 per cent

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[a]*Supra*, p. 57.

surviving, the terminal point of the short stub, to 45 per cent surviving. Fitting the data under these conditions, the remaining panel members to a man chose L_0, 22 average life curves.

On the other hand, one of the characteristics noted of experience predicted by an orthogonal polynomial analysis is delayed retirement at advanced ages. This in turn produces a survivor curve that tends to "tail out". Actually, life table values approach absolute zero per cent surviving asymptotically in this method because of the basic form of the expression governing the retirements. That is, the percentage surviving, y, at any age, X, is defined by orthogonal polynomials as

$$y = e^{-f(x)}$$

where $f(x)$ is a polynomial in terms of age. It is apparent from this equation that y cannot become zero until $f(x)$ becomes infinitely large.

If $f(x)$ is of the second degree, the effect of delayed retirement is, generally, not too noticeable because the per cent surviving figures fall to values which can be assumed to be zero at relatively early ages. Third degree expressions bring about this condition even sooner, of course. However, the L_0 type curves normally are fitted best by a polynomial of the first degree. In these instances the rate of retirement

does not increase as rapidly with age as do those of a higher order, and, hence, the older units remain in service somewhat longer.

A similar situation was encountered for the L_2 curves. The data for both life tables indicated delayed early retirements somewhat beyond those expected for an L curve. Yet, over-all, each experience agreed fairly well with this general type curve. The heavily stubbed versions of these data suggested high modal S or R type curves because of the lack of any early retirements. Only five of the 16 choices indicated for the heavily stubbed versions of these life tables were even L- types and they were all L_5. All service life estimates were underestimated as well.

The lack of early retirements caused the orthogonal analysis to develop data which missed the standard as well but the "tailing" characteristic brought them much closer to the correct version at the more advanced ages than those selected by any panel member.

A pronounced departure from a trend established by the data as a whole also entered into the difficulties encountered by the panel on the L_4 curves. The experience presented in life table No. 21 was unusual in that it indicated a considerable number of early retirements. This early behavior was followed by an extremely retarded retirement rate which

produced a plateau on the survivor curve and, finally, a
gradually increasing rate such as is normally found in the
L-types. The heavily stubbed version was terminated near
the end of the plateau, thus producing a so-called J or
hyperbolically shaped plot ending at about 77 per cent sur-
viving. This form is completely foreign to any of the Iowa
types. Three of the panel members who were asked to fit this
curve declined at first and the remaining men expressed
doubts as to the propriety of using the Iowa method. The
only way a fit could be made was to ignore the early re-
tirements. This obviously produced overestimates in all
cases.

A third degree retirement rate equation developed by
orthogonal polynomials defined experience which agreed quite
well with this particular curve shape. The versatility ex-
hibited by the method in fitting these data represents a
distinct advantage over the Iowa method.

In the case of these data as well as those previously
discussed the lightly stubbed versions indicated sufficient
L-type characteristics to allow the panel to make considerably
better estimates. In general, these longer stubs terminated
in the range of 40 to 50 per cent surviving.

Both curve fitting methods proved to be somewhat less
effective with the R-type data than they were with the left-
moded curves. It appears that this was due principally to

the contrasting retirement rates present in experience of this type. Characteristically, few retirements are noted until beyond the mean age. This period of relative inactivity is followed by one of very rapid removal of property from service which causes the survivor curve to drop almost vertically and to terminate with little or no tail.

The retirement rates over the first period decrease and those present in the second interval increase as the type designation passes from R_1 to R_5. This is of particular interest in the interpretation of the results reported for these data. Of the eight significantly better estimates indicated for this general mortality type, six were made by the panel and all of these occurred on the lower modal types, i.e., R_1, R_2, and R_3. In these instances, a sufficient number of retirements were realized over the early years to establish a trend characteristic of a long-lived property having a relatively small retirement rate throughout the entire life span. This is essentially the mortality experience predicted by the orthogonal polynomial analysis in each case. Thus, the tendency to describe experience which "tailed out" and, hence, which agreed well with L-type experience, proved to be undesirable for some of the right-moded data. In most all cases, this method of analysis overestimated both comparison moduli because of the failure to predict the rapid retirement rates in the later years.

These same curve characteristics were, no doubt, also responsible for the increased variation in the estimates reported by the panel for the right-moded experience over the L-types.[a] However, the established form of the standard curves prevented the extreme errors noted in the orthogonal analyses.

In the higher moded curves, $\underline{i.e.}$, R_4 and R_5, the influence of early retirements diminishes. Instead, the principal trait is the manner in which the retirement rate begins to increase rapidly from essentially zero at about the mean age. Under these conditions the orthogonal estimates showed some improvement because even the heavily stubbed data indicated to a certain extent the sharply increasing rate. If this retirement rate trend was well established in the stub data, the orthogonal analysis approximated right-moded types quite well. Moreover, the continually increasing rates effectively lessened the tendency for the estimated data to "tail out".

It is not certain whether the curve type alone was responsible for the significantly better estimates which the orthogonal analyses showed over the Iowa method on the R_4 and the R_5 data. The terminal point on the lightly stubbed curve of life table No. 46 (the short-lived R_4 experience)

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[a]$\underline{Infra}$, p. 99.

was at 33.32 per cent surviving and in the right margin of
the page, slightly beyond the grid lines. The preceding point
was plotted at 69.71 per cent. The extremely good estimates
reported by some of the panel members for this stub curve and
the relatively high values chosen by the others suggest the
possibility that the latter group did not notice the last
point because of its location. However, it is equally pos-
sible that those men chose to ignore the single low value be-
cause it was so far removed from the trend established by the
remaining data.

The results from the lightly stubbed R_5 data were par-
ticularly interesting because both methods did very well.
However, the estimates based upon the orthogonal polynomial
analyses agreed exactly with the average life standards for
both life tables. A deviation of one year in an estimate of
average life from any one of the 16 panel members would be
enough to cause the mathematical analyses to be significantly
better. Actually, the panel did about as well as could be
expected since eight men produced correct estimates, seven
missed the standard by only one year, and the final member
missed by eight years.

Contrary to the results observed for the left- and right-
moded types, little difference was noted in the estimates
made by either method on the symmetrically shaped curves. No
type characteristic which might cause a particular analysis

approach difficulty was observed.

Three significantly better estimates were reported for the S-type curves. Two were accorded to the Iowa method and one to an orthogonal analysis. For the S_0 experience the panel produced the better results principally because of the difficulty encountered by the orthogonal polynomial analysis of life table No. 43. An inspection of the lightly stubbed version of these data showed that the retirement rates increased in size slowly but almost uniformly over the time interval involved. As a result, the rates were fitted with a straight line expression having a gentle slope. Thus, the extrapolated rate values did not increase in size rapidly enough to cause the estimated life experience to terminate as did the standard data. Instead, the characteristic tail developed at the advanced ages and quite high estimates of the average life and the expectancy-average service life resulted.

The orthogonal analyses of the heavily stubbed S_3 curves were very good. These data were again higher moded plus the fact that the stubs were sufficiently long to indicate a few of the fairly large retirement ratios. The total effect of these factors indicated a curvilinear expression for the retirement ratios and thus, the rates extrapolated for the advanced age intervals were large enough to minimize the "tailing".

The few high retirement rates which seemed to aid the mathematical analysis of the short stub proved to be misleading to the panel. Actually, the stub for the long-lived data in particular had very pronounced right-modal characteristics. Six of the eight men who analyzed the data chose a type R_4 curve. However, the trend of high retirement rates was not continued beyond the short stub. Instead, the overall experience tended to agree quite well with the S_3 mortality pattern. This shift in the trend of the retirement rates was sufficiently noticeable in the lightly stubbed version to make the correct form more apparent to the panel. It is interesting to note that the orthogonal analysis of these same lightly stubbed data produced a poorer estimate than was reported for the heavily stubbed data. This difference was probably due to the changing trend in retirement rates noted above.

To summarize, on the basis of a comparison of estimated annual and accrued depreciation, little evidence was shown to support the conclusion that either method of retirement analysis was consistently better than the other. However, a fairly good indication of the possibility that some differences existed with respect to general mortality type or to particular situations was observed. That is, the Iowa method panel encountered some difficulty in recognizing the correct trends for the left-moded, heavily stubbed data, particularly

those in which the retirement rates did not increase very
rapidly. Estimated experience based upon an orthogonal
polynomial analysis, however, agreed fairly well with the
standard under these conditions. Likewise, the orthogonal
approach fairly consistently overestimated the comparison
moduli for the right-moded data. While the panel's estimates
were generally high as well for these data, they tended to be
somewhat closer to the standards, at least in the case of the
lower modal curves.

Some difficulty was experienced by the panel in matching
data which departed considerably from the forms established
in the standard curves. In some instances, e.g., experience
which "tailed out" or bimodal data, the orthogonal analysis
was seen to be more versatile than the Iowa method in handling
the data. On the other hand, the principal difficulty noted
in the use of the orthogonals, excluding the extensive calcu-
lations necessary to arrive at an estimate, was the tendency
to describe experience in which retirement at the advanced
ages was delayed unusually long. This occurred primarily on
the low modal curves.

The results taken as a whole do not tend to confirm the
opinion expressed in the 1943 report of the National Associa-
tion of Railroad and Utilities Commissioners that in the
analysis of stub data extending to approximately 50 per cent
surviving, the Iowa method would yield "fairly reasonable

estimates" of average life but that it would be "less satis-factory for accrued depreciation" (32).[a] Only seven of 60 tests conducted indicated that the orthogonal polynomial analysis produced a significantly better estimate than the Iowa method. Of these seven, only three were measured by accrued depreciation and all were based upon heavily stubbed life tables, i.e., terminal points at roughly 70 per cent surviving.

The Iowa method was likewise credited with three sig-nificantly better accrued depreciation estimates but two of these were from analyses of lightly stubbed data which ex-tended to the vicinity of 50 per cent surviving. It seems reasonable to conclude from this that generally little dif-ference exists between the estimates prepared by either method for annual or accrued depreciation.

A corollary to the above conclusion is that on the basis of the indications noted in the present investigation the analysis of retirement ratios rather than life table data produces no consistent significant difference in estimates of mortality dispersion or depreciation requirements. Instead, it appears that the specific instances in which the orthogonal polynomial analyses gave better results generally resulted from the mathematical nature of the approach. That is, the

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[a]_Supra_, p. 51.

features which tended to produce "tailed out" life experience or which permitted the fitting of bimodal data were characteristics of the mathematical expressions derived rather than original form of the data analyzed. The desirability of the mathematical approach could be further determined by comparing the results obtained in this study with the estimates for the same data based upon analyses by the Gompertz equation, for example, or the Gompertz-Makeham approach of the Bell System.

The Iowa Type Curves

Since each of the 60 stub curves was analyzed by eight panel members, considerable information was available for further study with regard to possible characteristics inherent in the use of the Iowa method. The tendencies investigated were limited to error and bias under various conditions of mortality dispersion, stubbing, and length of average service life. Each indication of influence was examined statistically on the basis of the appropriate variance ratio or t test. A 1 per cent level of significance was adopted for these comparisons and the corresponding values of F and t were taken from the work of Pearson and Hartley (35).

Error

Two indications of the extent of error present in the panel's estimates were determined. The first considered the general mortality patterns in which the error noted in the estimates for the lightly stubbed data was significantly less than observed in those made on the heavily stubbed curves. These tests were accomplished using the variance ratio of the within group mean square for the heavily stubbed data to within group mean square for the lightly stubbed versions. The results of these comparisons have been presented in Table 4.

The second determination attempted to quantify the amount of error or the extent of agreement achieved by the panel on the long and short stub curves. Since a relative measure of the variation was desired, the standard deviation of the heavily stubbed average life estimates as given by the square root of the within mean square value was divided by the average of the two corresponding standard average life values. Similar computations were completed for the other test data categories. This ratio approximated a coefficient of variation in that the smaller the value, the less the relative error or variation indicated. Values for the ratios as determined under each mortality type have been given in Table 4.

Table 4. Mortality types showing significant difference in error present in panel's estimates for heavily and lightly stubbed data

Comparison base	Mortality types
Average life	L_0, L_1, L_3, L_4, R_5, S_3
Expectancy-average service life summation	L_0, L_1, L_3

An inspection of these results reveals that the extent of stubbing significantly influenced the amount of error observed in the panel's estimates of left-moded data. Moreover, little or no influence was noted in the fittings reported for the right- and symmetrically-moded curves.

These contrasting indications can be traced principally to differences in the longer stubs. That is, the short stubs of most types did not decisively show specific mortality characteristics. The possible exceptions to this observation may have been the long-lived, higher modal R-types. Because of the indecisive trends represented in the stubs different type curve and average life choices could be selected for each set of data. However, the lightly stubbed, left-moded curves were fairly well defined. The relatively high early retirement rates had caused the survivor curves to indicate mortality trends which were hard to confuse. Consequently, the panel's selections were much more uniform for

these data.

With the possible exception of the high-moded R curves, the longer stubs for the right- and symmetrically-moded data did not show the same relative improvement in definition. Thus, the variation in the panel's estimates remained essentially the same for either the short or long stubs.

The consideration of the error present would not be complete without noting the quantitative relationships shown in Table 5. It is apparent from these data that while a significant difference was found in the error of the estimates reported for the left-moded, lightly stubbed curves over those for the heavily stubbed experience, somewhat better over-all agreement was found among the panel's choices for this general mortality pattern than for either right-moded or the symmetrical types. As might be expected the greatest error occurred in the fittings of the R-type curves while the results from the symmetrical estimates indicated a level of variation somewhere between the other two general types.

These tendencies were again most likely due to the better curve definition in the L-types. Left-moded data tends to exhibit relatively high retirement rates during the early years. Thus, it is generally found that an L-type survivor curve will quickly fall from the radix of 100 per cent surviving. This is quite a distinctive characteristic, particularly for the lower modal types, and it, therefore,

Table 5.   Variation coefficients indicated for panel's
estimates

General mortality type	Coefficient of variation, per cent			
	Average service life		Expectancy-average service life summation	
	Heavily stubbed	Lightly stubbed	Heavily stubbed	Lightly stubbed
L_0	8.1	2.6	6.6	2.5
L_1	7.9	3.9	9.5	4.6
L_2	4.3	4.8	4.9	8.4
L_3	32.6	4.5	26.8	10.3
L_4	38.2	9.2	7.8	16.3
R_1	18.4	10.7	11.0	16.3
R_2	18.5	11.4	12.2	16.5
R_3	15.1	29.3	10.3	13.3
R_4	12.1	15.0	13.1	23.3
R_5	60.4	8.8	18.7	16.7
S_0	13.5	8.6	11.9	12.5
S_1	8.3	7.2	11.5	9.0
S_2	10.7	6.6	16.3	10.8
S_3	19.9	6.8	10.2	11.8
S_4	9.2	8.2	7.8	14.8

tends to minimize the doubt as to the retirement trend
established. This, however, does not mean that the indicated
trend will be correct or that it will continue. The contrary
has been fairly well illustrated in the results from the
comparison of the Iowa method and the orthogonal polynomials
as well as in those discussed below under bias. But the fact
that a definite trend is established fairly early improves

the possibility of agreement among analysts considerably.

Over-all, the other general mortality types have discriminating traits as well but, unlike those of the L-types, these features are not as pronounced in the early years. Hence, there is a possibility that analysts will differ in what tendencies they recognize in a stub particularly if the data do not extend somewhat beyond 50 per cent of the radix.

It is not possible to compare the results of these tests for error with those originally reported by Winfrey (48, p. 88-89) because the method of procedure as well as analysis was different. However, the data from the present investigation tend to support his conclusion that the amount of error decreases as the stub increases, at least for left-moded experience. Further, an examination of his results revealed that, in general, less variation was noted in the fittings made on the left modal data than on the other two general types. The same tendency was observed in the findings reported in this dissertation.

Bias

An interesting set of results was obtained from an analysis performed to determine whether the group of eight fittings reported for any particular stub represented a systematic bias. The algebraic sum of the deviations determined from the standard for each comparison base was tested to see

if it differed significantly from zero. That is, if every panel member selected the proper average life and type curve, the difference between the standard and the comparison modulus for each fitting would be zero and, hence, so would be the sum of the deviations. The significant findings resulting from these determinations have been summarized in Table 6.

It is apparent from an inspection of these results that with one exception every indication of systematic bias for the left-moded data was negative, that is, due to under-estimation. The exception occurred for the long-lived L_4 experience. As noted previously, this life table was bimodal, having a very high number of early retirements as well as an equally high rate of retirement over a later time interval. Since the resulting survivor curve was completely foreign to all the Iowa type curves, a close fit from the panel was not possible. Any selection of a type curve had to be accomplished by essentially ignoring the early retirements.

It is apparent that for any group of survivor curves which contains a consistently larger number of bimodal experiences the Iowa type curve method may not produce as satisfactory results as were observed in this investigation. While it is believed the original life tables used in the present study were quite representative, the difficulty encountered by the panel with the bimodal data would indicate

Table 6. Indications of systematic bias in panel's estimates

General mortality type	Sign of systematic bias found[a]			
	Average service life		Expectancy-average service life summations	
	Heavily stubbed	Lightly stubbed	Heavily stubbed	Lightly stubbed
Long life data				
L_0	0	-	0	0
L_1	-	-	-	-
L_2	-	-	-	-
L_3	0	-	0	0
L_4	+	+	+	+
R_1	+	0	+	0
R_2	+	+	+	+
R_3	+	0	+	+
R_4	+	0	+	0
R_5	+	0	0	0
S_0	-	0	0	0
S_1	-	+	0	0
S_2	+	0	0	0
S_3	0	+	0	-
S_4	+	+	+	0
Short life data				
L_0	-	-	-	-
L_1	-	-	0	-
L_2	-	-	-	-
L_3	0	0	0	-
L_4	0	0	-	0
R_1	0	0	0	0

[a]Symbols: 0 -- no systematic bias; - -- underestimated; + -- overestimated.

Table 6. (Continued)

| General mortality type | Sign of systematic bias found | | | |
| | Average service life | | Expectancy-average service life summations | |
	Heavily stubbed	Lightly stubbed	Heavily stubbed	Lightly stubbed
R_2	0	0	-	0
R_3	+	0	+	+
R_4	0	+	0	+
R_5	0	0	+	0
S_0	0	0	0	0
S_1	0	0	-	0
S_2	0	0	0	0
S_3	0	0	0	+
S_4	0	0	0	+

that a survey to determine the occurrence frequency of these special forms would be in order. Moreover, if the results indicated the need, perhaps additional standard type curves could be developed to analyze these data.

The tendency of the panel as a group to estimate low was likely due to two principal factors. They were, first, the difficulty experienced by the members in recognizing left-modal characteristics in some of the short stubs, and, second, the prevalence of standard data which "tailed out" at advanced ages. The latter factor, in particular, suggests that a study might be conducted to determine whether the left-modal Iowa curves, as they are now defined, might

terminate too quickly.

The problem presented by the need for smaller retirement rates at advanced ages may be minimized for the lower modal curves when the work of Couch becomes available (12). He has classified a number of the so-called J-shaped curves into an O-type which will supplement the present Iowa type curve set. The two predominate characteristics of these particular data are, first, a high number of early retirements and, second, delayed retirements during the later years. The maximum slope of the survivor curve occurs at the origin, thus, the designation, O-type.

In contrast to what occurred for the left-moded experience, every indication of systematic bias for the right-moded was positive except for a single instance. It appears that the primary cause for this group tendency to overestimate was the difficulty in determining from the stub data the age at which the characteristically large increase in retirement rate would occur.

Not only were fewer indications of a systematic bias noted in the data reported for the symmetrical curves than for the other types but also the biases observed were fairly evenly distributed between the positive and negative tendencies. Thus, no specific conclusions with regard to the general mortality type were indicated.

Further analyses were run to determine whether a

significantly different bias was present in the fittings of
short-lived experience than in those reported for long-lived
data. This was done in an effort to learn more concerning
the nature and cause of the biases found in the panel's work.
Two separate tests were needed for this particular study be-
cause of the method chosen to group the experimental data.
One involved the biases in long- and short-lived experiences
under heavy stubbing conditions and the other for the lightly
stubbed versions of these same data.

For both analyses the estimate of statistical signifi-
cance was based upon the variance ratio of the between group
mean square to the within group mean square. The findings
from these tests are presented in Table 7.

As might be expected some indication of significantly
different biases in the long- and short-lived fittings were
shown, although the data were not altogether conclusive.
That is, the means of the estimates for the short-lived,
heavily stubbed curves generally tended to fall closer to the
correct values than those of the long-lived properties, but
only a few significantly so as seen from Table 7. Further,
in some cases (L_0 and L_4) this arrangement of the means with
respect to the standard was reversed and the mean of the
estimates based upon short-lived properties was shown to be
significantly more biased. However, these particular occur-
rences were due, normally, to a considerable departure of the

Table 7. Mortality types for which significantly different
biases were noted in panel's estimates of long-
and short-lived data

Comparison base	Mortality types
Average life, heavy stubbing	L_0, L_2, L_4, R_1, R_2, R_4, S_0, S_4
Average life, light stubbing	L_3, L_4, R_2, R_4, S_1
Expectancy-average service life summation, heavy stubbing	L_0, L_1, L_2, L_4, R_2, R_4, R_5, S_0
Expectancy-average service life summation, light stubbing	L_0, L_1, L_2, L_4, R_2, R_4

short-lived standard data from the Iowa type experience.
This condition has been discussed previously.

An exception of considerably more interest was found for
the R_5 experience. It will be recalled that the short-lived
versions of three generalized mortality types, S_2, S_4 and
the R_5, were synthesized from the corresponding long-lived
life tables by plotting the data to a reduced scale.[a] Of the
six comparisons made on the estimates for these particular
data only one, the heavily stubbed, short-lived R_5 curve,
showed the biases in the fittings of the two versions to be
significantly different. However, the long-lived data was

[a]Supra, p. 56.

biased less than the shorter, synthesized experience. An in-
spection of the longer-lived data indicated that they con-
formed very closely to the general right-moded pattern while
the other version lost many of the original characteristics
in the heavily stubbed re-plotting. Thus, this set of syn-
thesized data appeared to be fitted equally well by L-, R-,
or S-type curves, all of which indicated about the same
average life but described somewhat different mortality dis-
persions.

In general, a situation similar to that described for
the heavily stubbed curves was found for the lightly stubbed
data except that even fewer instances of significance were
discovered. Thus, while the panel's estimates for the short-
lived data appeared to be less biased than the longer-lived
experiences under heavy stubbing, this tendency was reduced
slightly when the stub curves were extended to the lightly
stubbed form.

The results from the tests just described indirectly
suggested that the extent of stubbing present in the data may
have an effect on the bias present in the panel's choices but
that it may not be as great as would normally be expected.
Unfortunately, no completely satisfactory tests for the bias
resulting from stubbing could be devised from the experimental
data. It was felt that it would not be appropriate to submit
stub curves of different lengths from a common life table to

the same panel member because of the possibility of data recognition. Nevertheless, some indication of the effect of stubbing was deemed to be desirable. Therefore, an analysis of the results was specified which compared the fittings made on the heavily stubbed version of an experience with those for the lightly stubbed form. It should be recalled, however, that one group of eight panel members fitted, for example, the former version of the data while the other eight worked with the latter. This analysis approach, admittedly, has a doubtful element present since it measures any difference in the results on the basis of the work of two different groups of panel members. It is believed, however, that this particular feature did not influence the results to a great extent.

The mechanics of these tests for any mortality type involved comparing the variance ratio of the correction for the mean to the within mean square based upon both the long- and short-lived data. The correction had one degree of freedom while the over-all term had 14. Significant findings have been presented in Table 8.

Surprisingly few indications of a significant effect due to the extent of stubbing were shown. If any relationships existed between the amount of stubbing and mortality type, they were not indicated by these particular tests. In each of the curve types listed in Table 8 the differences in the

Table 8. Mortality types for which the panel's estimates were affected significantly by the amount of stubbing

Comparison base	Mortality type
Average life	L_2, L_4, R_5, S_1
Expectancy-average service life summation	L_2, R_5, S_1

results appeared to be due principally to better defined mortality characteristics, i.e., smoother data, in the longer stubs.

In retrospect it is believed that the analyses to show the relationships of biases to average age or stubbing under various mortality dispersions were not completely satisfactory. That is, frequently it appeared that any indication of a trend was substantially dampened by the fittings reported for one of the curves or even a single stub under a general curve type. Almost invariably this occurred in cases where the exceptional curve either departed considerably from an Iowa type or it differed somewhat from the other experience in the extent as well as the clearness of mortality characteristics shown by the data.

This condition existed because the original retirement experience was selected with the basic purpose of this investigation in mind, that is, to compare the Iowa type curve

method with the use of orthogonal polynomials. No particular consideration was given to the relative traits, e.g., smoothness of the data, of curves selected for each type. Rather, it was desired to obtain a fairly wide range of conditions so that each method would receive a thorough test.

It is felt, therefore, that a subsequent study performed with data selected to minimize the difficulties just discussed would be desirable. A suggested procedure might be to utilize a panel of engineers once more and have this group fit data for which the various versions of average life and stub length under any mortality type would all be synthesized from a single life table much in the same manner used for the S_2, S_4, and the R_5 experiences in the present investigation. It is further suggested that the entire set of curves not be submitted to the panel at one time. If the test data would be divided into three or four groups and only one set fitted at any one time, it is felt that stubs of different lengths but from the same retirement experience could be included in the test without much chance of data recognition.

The information to be gained from a study such as the one just described would tend to supplement evidence presented in this dissertation. It would either confirm the indications noted herein that the amount of stubbing or the length of average service life fails to significantly influence the bias of mortality estimates or it would provide

additional insight into the nature of the relationships
present.

CONCLUSIONS

The analyses of the mortality dispersion estimates reported in this dissertation disclosed a number of significant indications, the most pertinent and valid of which are summarized below in the form of specific findings or general conclusions:

1. Under the conditions adopted, i.e., the stipulations for the analysis of the retirement data, the standards assumed, and the comparison bases used, no consistent superiority was enjoyed by either the Iowa type curve method or the use of orthogonal polynomials in estimating mortality dispersion.

2. Based upon the over-all results of this limited study as indicated above no consistent significant difference in results appears to be attained in basing dispersion estimates on the analysis of retirement ratios as opposed to the use of life table data.

3. The method of orthogonal polynomials showed some advantage in handling experience that could be termed abnormal on the basis of the standard Iowa types, e.g., bimodal data, or a low rate of increase in the size of the retirement ratios at advanced ages. This indication was particularly

noticeable under the extreme stubbing in case of the accrued depreciation comparison base.

4. The Iowa type curve method tended to produce better estimates of dispersion for the lower modal R- and S-type experiences than did the orthogonal polynomials because of the latter method's failure to predict the relatively rapid termination of experience which is characteristic of these types, particularly the right-moded data.

5. The amount of error present in the panel's estimates of left-moded data was, in most instances, significantly reduced as the length of the stub curve was increased. No general trend of this type was observed in the panel's estimates for the other two basic mortality patterns.

6. The agreement attained by panel members in their analyses of a particular experience was greatest in the case of the left-moded data and least for the right-moded curves.

7. With one exception all indications of systematic bias in the panel's estimates of left-moded experience were negative, i.e., due to underestimation.

8. With one exception all indications of systematic bias in the panel's estimates of right-moded experience were positive, i.e., due to overestimation.

9. No indication of consistent significant effect upon the bias of the estimates could be found due to the length of average service life encountered.

10. The incidence of significant effects due to stubbing in the bias of the panel's estimates was considerably less than would normally be expected.

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ACKNOWLEDGMENT
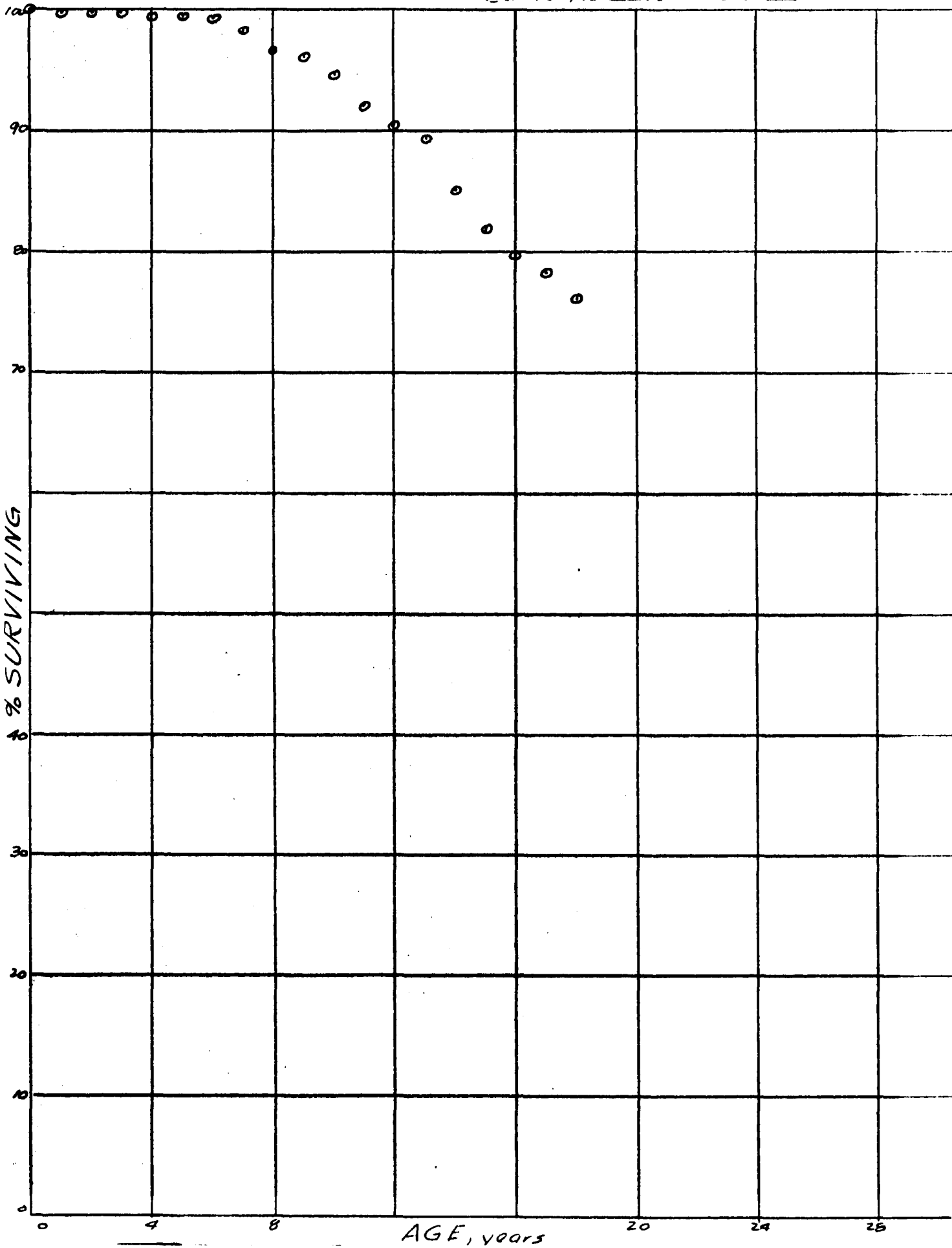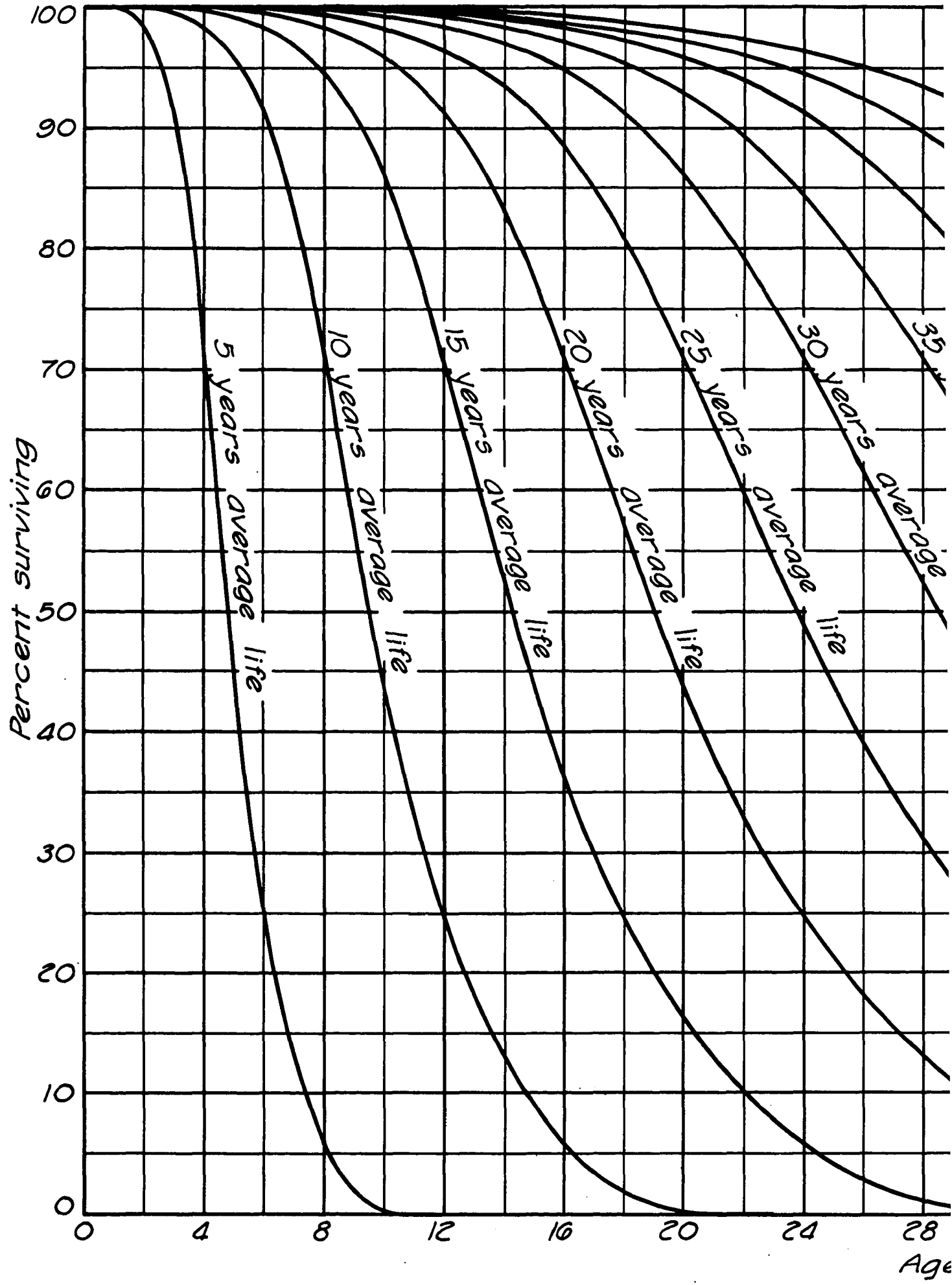
APPENDICES

APPENDIX A: IOWA TYPE CURVE PANEL MEMBERS

1. Mr. C. Radford Berry, Chief Valuation Engineer
   Pennsylvania Public Utility Commission
   Harrisburg, Pennsylvania

2. Dr. Barney Bissinger, Department Head,
   Department of Mathematics, Lebanon Valley College
   Annville, Pennsylvania

3. Mr. D. H. Callen, Rate and Valuation Engineer,
   Pioneer Service and Engineering Company
   231 South LaSalle Street, Chicago 4, Illinois

4. Mr. Morris R. Carlson
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   600 North Second Street
   Harrisburg, Pennsylvania

5. Mr. William H. Caunt, Jr., Chief Depreciation Accountant
   Public Service Electric and Gas Company
   80 Park Place
   Newark 1, New Jersey

6. Dr. W. Chester Fitch
   Gannett, Fleming, Corddry and Carpenter, Inc.
   600 North Second Street
   Harrisburg, Pennsylvania

7. Mr. Francis S. Haberly, Engineer
   122 South Michigan Avenue
   Chicago 3, Illinois

8. Mr. James F. Haley, Senior Utilities Engineer
   Public Utilities Commission, State of California
   California State Building
   San Francisco 2, California

9. Lt. Col. Jean C. Hempstead, Department of Mathematics
   United States Air Force Academy
   Lowry Air Force Base
   Denver, Colorado

10. Mr. Russell L. Howard
    The Phillips Petroleum Company
    Bartlesville, Oklahoma

11. Mr. Winn Jones, Depreciation Engineer
    Columbia Gas System Service Corporation
    120 East 41st Street
    New York 17, New York

12. Mr. Con L. Lindholm, Property Records Supervisor
    Iowa-Illinois Gas and Electric Company
    Davenport, Iowa

13. Mr. James P. McKean
    American Appraisal Company
    525 East Michigan Street
    Milwaukee 1, Wisconsin

14. Dr. J. P. Mills, Associate Professor
    Industrial Engineering Department
    Iowa State College
    Ames, Iowa

15. Mr. Henry R. Paterick
    Bureau of Public Roads
    United States Department of Commerce
    Washington, D. C.

16. Mr. Robley Winfrey, Chief of Personnel and Training
    Bureau of Public Roads
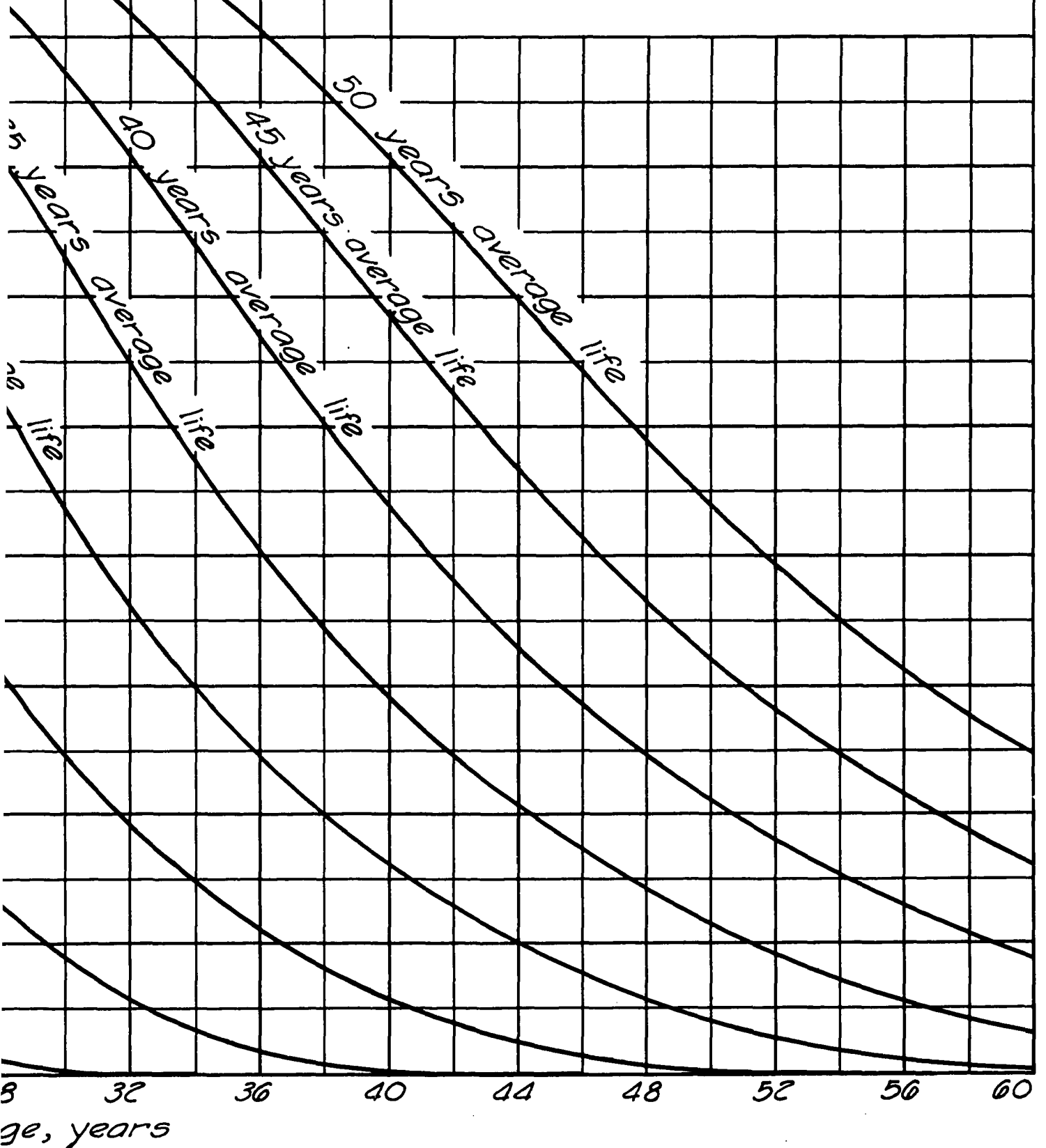   United States Department of Commerce
    Washington, D. C.

APPENDIX B: EXAMPLES OF MATERIAL SUBMITTED TO PANEL

Curve No 45.   Code C

% SURVIVING

AGE, years

L_3 Type Survivor Curve
from Bulletin 125
Iowa Engineering Experiment Station
Iowa State College

50 years average life

45 years average life

40 years average life

35 years average life

30 years average life

| 8 | 32 | 36 | 40 | 44 | 48 | 52 | 56 | 60 |

ge, years

General Information and Instructions Concerning the Stub Curves

The thirty stub curves which have been included in this envelope have been selected from approximately fifty complete, annual rate survivor curves. The data represented are mainly from electric and gas utility accounts. Brief descriptions of the properties involved have been included so that panel members may have some further bases upon which to exercise their judgment as to the appropriateness of their curve selections. The Federal Power Commission Uniform System of Account numbers have been given where it was possible. These numbers have been preceded by "Electric Utility," or "Natural Gas" in parentheses to designate from which system the account numbers were taken. In addition, the vintages or placement years of the properties as well as the annual rate experience band years have been indicated.

Referring once more to the curves, it will be seen that they have been reproduced on sheets of tracing paper somewhat smaller than Codex 418 graph paper. The reproduction was felt necessary because of the consistency factor and the shorter paper was dictated by the economy of the situation. Each curve was plotted on Codex paper then traced as accurately as possible on multilith paper plates. It is felt that this method of presentation will have little or no effect on the results of the experiment.

Each curve has a number as well as a code letter. The former refers to the account and the latter to the length of the stub.

Two sets of Iowa type curves have been included. The Iowa Engineering Experiment Station set presents the curves in intervals of five years in average life. This is the original version of the curves as prepared under the direction of Robley Winfrey. The consulting firm of Gannett, Fleming, Corddry and Carpenter of Harrisburg, Pennsylvania, very generously offered to provide us with copies of their more detailed curves. The original drawings for both of these sets were accurately made on Codex paper but in printing the paper shrinks slightly. So, it may be that with some curves types the grid lines may not exactly line up. The difference should not be so great that compensation could not be made for this in the fitting.

There is no specified way or order in which you fit the curves as we would like you to use the methods to which you are most accustomed. Either or both of the type curve sets may be used. We would merely like to have you record on the attached summary sheet what you feel to be the most appropriate Iowa type curve for each stub and your corresponding estimate of average life based upon the points presented.

Two additional pieces of information are desired. They are, first, your estimate of the time spent fitting the curves, and second, a brief summary of your experience with and your use of the Iowa type curves. There are spaces on the summary sheet for these answers, also. Our purpose in gathering these data is, merely, to aid in the interpretation of the results obtained from the experiment.

When you have fitted all the curves to your satisfaction, will you please return the data summary sheets and the Gannett, Fleming, Corddry and Carpenter Iowa type overlays. A return envelope has been provided for your convenience. You may keep, if you like, the stub curves and the Iowa Engineering Experiment Station set of curves.

Description of Property
Included in Accounts

Curve No. 2:   FPC (Natural Gas) Acc't. No. 373, Automobiles and Trucks. Vintages: 1925-1950, annual rate experience band: 1939-1950.

Curve No. 4:   FPC (Electric Utility) Acc't. No. 316, Miscellaneous Power Plant Equipment such as air compressors, communication system, cranes, etc. Vintages: 1904-1954, annual rate experience band: 1950-1954.

Curve No. 6:   FPC (Natural Gas) Acc't. No. 358, Structures and Improvements (permanent buildings and land improvements). Vintages: 1861-1951, annual rate experience band: 1939-1951.

Curve No. 8:   FPC (Electric Utility) Acc't. No. 315, Accessory Electric Equipment such as auxiliary generators including boards, switching equipment, control equipment. Vintages: 1899-1954, annual rate experience band: 1950-1954.

Curve No. 10: FPC (Natural Gas) Acc't. No. 352, Pumping Station Equipment --auxiliary equipment. Annual rate experience band: 1939-1951.

Curve No. 13: FPC (Electric Utility) Acc't. No. 312, Boiler Plant Equipment. Vintages: 1906-1954, annual rate experience band: 1950-1954.

Curve No. 14: FPC (Natural Gas) Acc't. No. 377, Tools and Equipment such as air compressors, including driving unit and vehicle. Vintages: 1924-1949, annual rate experience band: 1938-1949.

Curve No. 15: FPC (Natural Gas) Acc't. No. 378, Communication Equipment. Vintages: 1900-1951, annual rate experience band: 1939-1953.

Curve No. 17: FPC (Natural Gas) Acc't. No. 362, Meters. Vintages: 1880-1953, annual rate experience band: 1939-1953.

Curve No. 18: FPC (Natural Gas) Acc't. No. 373, Transportation Equipment, ½ to 1½ ton trucks. Vintages: 1924-1949, annual rate experience band: 1938-1949.

Curve No. 20: FPC (Electric Utility) Acc't. No. 379, Miscellaneous Equipment such as recreational, hospital, and restaurant equipment. Vintages: 1903-1949, annual rate experience band: 1920-1949.

Curve No. 21: FPC (Natural Gas) Acc't. No. 359, Mains--steel. Vintages: 1914-1949, annual rate experience band: 1920-1949.

Curve No. 22: FPC (Natural Gas) Acc't. No. 333.2, Field Compressor Station Equipment. Vintages: 1919-1952, annual rate experience band: 1922-1952.

Curve No. 23: FPC (Electric Utility) Acc't. No. 372, Office Furniture-- Mechanical such as typewriters. Vintages: 1910-1950, annual rate experience band: 1939-1950.

Curve No. 26:   FPC (Electric Utility) Acc't. No. 373, Transportation Equipment--Garage including battery chargers, brake linings, gasoline pumps, etc. Vintages: 1924-1954, annual rate experience band: 1950-1954.

Curve No. 27:   FPC (Natural Gas) Acc't. No. 332.2, Producing Gas Wells--Well Equipment--Pipe. Vintages: 1883-1950, annual rate experience band: 1939-1950.

Curve No. 28:   FPC (Natural Gas) Acc't. No. 377, Tools and Work Equipment such as portable air compressors, welders. Vintages: 1924-1949, annual rate experience band: 1938-1949.

Curve No. 29:   FPC (Natural Gas) Acc't. No. 362, Meters. Vintages: 1880-1954, annual rate experience band: 1939-1954.

Curve No. 31:   FPC (Electric Utility) Acc't. No. 312, Boiler Plant Equipment. Vintages: 1903-1949, annual rate experience band: 1920-1949.

Curve No. 34:   FPC (Natural Gas) Acc't. No. 359, Mains--Wrought Iron. Vintages: 1859-1949, annual rate experience band: 1920-1949.

Curve No. 36:   FPC (Natural Gas) Acc't. No. 331.1, Gas Well Structures. Vintages: 1914-1944, annual rate experience band: 1939-1953.

Curve No. 37:   FPC (Electric Utility) Acc't. No. 314, Turbo-Generator Units including main turbine driven units and accessory equipment used in generating electricity by steam. Vintages: 1910-1954, annual rate experience band: 1950-1954.

Curve No. 39:   FPC (Natural Gas) Acc't. No. 342, Storage Structures--Underground Storage Well Construction. Annual rate experience band: 1939-1954.

Curve No. 41:   FPC (Electric Utility) Acc't. No. 377, Tools and Work Equipment--Tractors and Ditchers. Vintages: 1924-1949, annual rate experience band: 1938-1949.

Curve No. 42:   Telephone Company, motor vehicles - classes 2, 3, 4, 5, 6, 7, 8, 12, 13. Vintages: 1932-1954, annual rate experience band: 1950-1954.

Curve No. 43:   FPC (Natural Gas) Acc't. No. 360, Pumping and Regulating Equipment--Rotary Positive Displacement Gas Pumps and Installations. Vintages: 1900-1949, annual rate experience band: 1920-1949.

Curve No. 45:   FPC (Electric Utility) Acc't. No. 343, 352, Station Equipment--Distribution and Transmission. Vintages: 1906-1954, annual rate experience band: 1950-1954.

Curve No. 46:   FPC (Natural Gas) Acc't. No. 321, Coal, Coke, and Ash Handling Equipment--steel. Vintages: 1892-1949, annual rate experience band: 1920-1949.

Curve No. 47:   FPC (Natural Gas) Acc't. No. 360, Pumping and Regulating
Equipment--Centrifugal Gas Pumps and Installations. Vintages:
1919-1949, annual rate experience band: 1920-1949.

Curve No. 50:   FPC (Natural Gas) Acc't. No. 36, Pumping and Regulating
Equipment, includes pressure gauges, timers, etc. Vintages:
1899-1949, annual rate experience band: 1920-1949.

Summary Sheet

Curve	Code Letter	Iowa Type	Average Life	Approx. Time to Fit Curve	Curve	Code Letter	Iowa Type	Average Life	Approx. Time to Fit Curve
No. 2					No. 27				
No. 4					No. 28				
No. 6					No. 29				
No. 8					No. 31				
No. 10					No. 34				
No. 13					No. 36				
No. 14					No. 37				
No. 15					No. 39				
No. 17					No. 41				
No. 18					No. 42				
No. 20					No. 43				
No. 21					No. 45				
No. 22					No. 46				
No. 23					No. 47				
No. 26					No. 50				

Brief Summary of your experience with the use of the Iowa curves:

APPENDIX C: ESTIMATED COMPARISON MODULI

Table 9a. Predicted type curves and average service lives, left modal data[a]

Panel member No.:	Type L_0 10[b]	27[c]	Type L_1 50	23	Type L_2 36	28	Type L_3 34	41	Type L_4 21	42
1	St-34	L_0-21	L_2-19	S_1-14	R_5-19	L_5-9	R_4-30	L_1-15	L_0-30	L_4-8
2	L_0-33	S_0-16	S_1-19	S_1-14	R_5-18	L_5-9	R_5-26	L_2-14	L_0-25	R_4-8
3	St-34	S_0-16	R_3-15	L_2-14	S_6-16	S_5-9	R_3-40	L_3-11	St-25	R_5-5
4	L_0-35	S_0-17	L_2-19	L_2-14	S_5-18	L_5-8	R_5-27	S_2-11	R_2-30	S_3-9
5	St-34	R_1-16	S_2-19	L_1-17	R_5-18	S_5-8	R_2-50	S_3-10	R_1-30	S_4-8
6	St-33	S_0-15	L_2-19	S_1-14	S_6-17	S_5-8	S_6-24	R_3-10	L_0-23	R_4-8
7	L_0-36	S_0-16	R_3-16	S_2-12	L_5-17	L_5-8	L_5-26	L_5-5	L_0-25	L_4-7
8	R_1-27	S_0-16	L_2-19	S_1-14	S_5-18	S_4-9	S_5-26	R_4-9	R_3-45	L_4-8
Orthogonal polynomials	32	21	18	18	22	9	31	17	16	8
Standard	33	24	22	16	23	12	30	11	17	9

[a]Heavily stubbed data

[b]Long-lived data

[c]Short-lived data

Table 9b. Predicted type curves and average service lives, left modal data[a]

Panel member No.:	Iowa type curve and average service life, years									
	Type L_0		Type L_1		Type L_2		Type L_3		Type L_4	
	10	27	50	23	36	28	34	41	21	42
9	St-31	L_0-22	L_2-20	L_2-15	L_4-20	L_5-10	S_5-25	R_4-10	L_1-22	S_3-9
10	St-31	L_0-22	L_2-22	S_1-14	L_3-23	R_4-11	R_4-28	R_4-10	R_1-19	R_4-8
11	St-31	L_0-22	S_1-20	R_2-14	R_4-20	L_4-11	L_4-29	L_4-10	St-23	L_4-9
12	L_0-32	L_0-22	L_2-20	S_1-14	S_3-21	S_3-11	S_4-28	S_3-11	R_1-20	L_4-8
13	L_0-32	L_0-22	S_1-19	S_1-13	S_3-20	S_4-10	S_4-28	R_3-10	L_0-22	R_4-8
14	L_0-30	L_0-22	S_1-19	S_1-14	S_3-20	L_4-11	S_4-27	S_3-10	S_0-19	L_4-9
15	L_0-33	L_0-22	S_1-20	S_1-14	S_5-20	S_3-10	R_5-27	S_2-11	R_1-19	R_4-8
16	L_0-33	L_0-22	R_2-19	S_1-14	S_3-20	S_2-11	R_4-27	L_3-11	S_2-22	L_4-9
Orthogonal polynomials	36	23	23	14	23	13	33	11	19	9
Standard	33	24	22	16	23	12	30	11	17	9

[a]Lightly stubbed

Table 10a. Predicted type curves and average service lives, right modal data[a]

Panel member No.:	Iowa type curve and average service life, years									
	Type R_1		Type R_2		Type R_3		Type R_4		Type R_5	
	15^b	22^c	13	18	8	6	4	46	37	26
1	St-35	L_0-17	R_2-35	S_2-6	S_0-45	L_2-58	R_3-40	S_3-30	R_3-40	L_2-28
2	L_0-33	R_1-15	S_0-43	L_3-7	S_0-46	R_3-55	R_3-42	R_3-33	R_3-42	R_3-24
3	St-35	St-18	L_2-30	R_4-6	S_0-45	R_3-52	R_2-50	L_4-29	R_3-40	R_3-22
4	L_0-40	St-18	S_0-41	L_4-6	S_0-47	S_1-60	R_2-50	S_2-34	R_2-55	S_1-27
5	R_1-25	L_0-18	R_1-45	S_2-6	R_2-35	S_1-62	R_2-50	S_3-29	R_3-45	S_0-37
6	St-35	R_1-14	L_1-40	R_4-5	S_0-46	S_1-60	R_3-40	S_3-29	R_3-45	S_0-39
7	LO-35	St-18	S_1-40	S_4-6	S_1-47	S_1-62	S_1-50	S_4-26	R_1-100	R_3-19
8	R_1-25	L_0-17	R_2-35	S_1-7	R_2-36	L_1-76	R_3-42	R_5-25	R_3-44	L_2-27
Orthogonal polynomials	32	14	67	9	53	81	41	44	45	9
Standard	26	16	32	6	30	51	36	31	33	17

[a]Heavily stubbed

[b]Long-lived data

[c]Short-lived data

Table 10b. Predicted type curves and average service lives, right modal data[a]

Panel member		Iowa type curve and average service life, years									
		Type R_1		Type R_2		Type R_3		Type R_4		Type R_5	
	No.:	15	22	13	18	8	6	4	46	37	26
9		R_1-25	L_0-19	S_0-40	S_2-6	R_2-36	S_1-70	R_4-35	S_3-35	R_5-32	S_1-25
10		R_1-24	L_0-18	L_1-40	R_3-6	R_2-35	S_0-78	R_4-35	R_3-32	R_5-33	R_4-16
11		St-30	St-18	L_1-43	L_5-5	L_1-44	L_1-86	S_3-37	L_1-50	S_6-33	S_4-16
12		R_1-25	R_1-15	L_1-42	L_3-7	R_2-36	S_1-60	L_3-40	S_1-43	R_5-33	S_5-17
13		L_0-31	R_1-15	S_0-40	R_4-7	S_0-42	R_2-66	L_3-40	R_2-40	R_5-32	R_5-17
14		R_1-25	R_1-15	S_0-39	S_2-7	R_2-35	L_2-64	L_4-36	R_4-31	R_5-32	R_4-16
15		R_1-25	L_0-18	R_2-33	R_3-7	R_2-34	L_1-42	R_3-36	L_1-46	R_5-33	R_5-17
16		R_1-25	R_1-15	S_0-40	L_2-7	R_2-35	S_0-38	R_5-34	S_1-39	R_5-33	R_4-16
Orthogonal polynomials		27	23	39	7	36	94	36	33	33	17
Standard		26	16	32	6	30	51	36	31	33	17

[a]Lightly stubbed

Table 11a. Predicted type curves and average service lives, symmetrical data[a]

Fitting method panel member	No.:	Iowa type curve and average service life, years									
		Type S_0		Type S_1		Type S_2		Type S_3		Type S_4	
		43^b	47^c	45	2	31	20	17	14	29	39
1		L_2-20	S_1-16	S_1-25	S_1-7	S_1-32	S_0-16	R_4-48	L_3-11	S_2-64	S_2-16
2		R_2-19	S_0-22	S_1-26	S_2-6	S_1-32	R_3-14	R_4-49	L_3-11	R_4-52	L_4-15
3		R_2-19	L_2-19	L_2-25	R_2-7	L_2-31	S_1-16	S_3-50	R_4-10	L_3-60	L_3-15
4		S_1-20	L_1-23	S_1-26	R_2-6	L_2-32	R_3-12	L_4-50	L_4-10	S_2-64	S_2-16
5		R_2-20	L_0-28	R_3-20	S_1-6	R_3-25	S_1-16	R_4-50	S_3-10	R_3-60	S_1-15
6		L_1-24	L_1-20	S_1-26	R_2-6	S_1-31	S_1-15	R_4-48	R_5-9	S_2-64	R_3-15
7		L_1-23	L_1-23	S_1-25	L_1-7	S_1-33	L_1-19	R_4-25	S_3-10	S_2-66	S_2-16
8		L_1-24	R_2-18	S_1-25	R_2-7	S_1-31	S_1-16	R_4-48	L_5-8	S_2-64	S_2-16
Orthogonal polynomials		23	22	29	7	33	17	53	9	63	15
Standard		26	19	27	8	28	14	52	9	55	14

[a]Heavily stubbed

[b]Long-lived data

[c]Short-lived data

Table 11b. Predicted type curves and average service lives, symmetrical data[a]

Fitting method panel member	No.:	Iowa type curve and average service life, years									
		Type S_0		Type S_1		Type S_2		Type S_3		Type S_4	
		43	47	45	2	31	20	17	14	29	39
9		R_1-30	L_1-20	S_1-27	R_2-7	L_2-31	L_2-15	R_4-50	R_4-9	L_3-61	L_3-15
10		R_1-26	R_2-18	R_2-29	R_2-7	R_3-28	L_2-15	L_3-56	R_4-9	R_4-54	S_2-15
11		L_1-28	R_2-17	S_1-28	L_1-8	R_3-28	S_3-18	S_3-53	L_4-9	L_3-63	S_2-15
12		L_0-31	R_3-17	L_1-32	R_2-7	S_1-30	L_2-16	S_3-53	S_3-10	L_3-62	L_3-16
13		S_0-25	R_1-20	S_1-28	R_2-7	S_1-30	L_2-14	S_3-53	L_4-10	S_3-57	R_4-13
14		S_0-25	L_2-20	S_1-28	R_2-7	S_2-28	L_2-15	R_4-50	L_3-11	R_4-55	L_3-15
15		S_0-26	S_0-19	S_1-27	R_1-9	L_2-31	S_1-15	L_3-57	R_4-10	L_3-62	S_2-16
16		S_0-25	R_2-18	S_1-28	L_1-8	R_3-27	S_1-15	L_3-57	R_4-9	R_4-54	S_2-15
Orthogonal polynomials		36	18	28	8	29	15	54	10	58	15
Standard		26	19	27	8	28	14	52	9	55	14

[a]Lightly stubbed

138

Table 12a. Expectancy-average service life summations, left modal data[a]

Fitting method panel member	No.	Type L_0 10[b]	27[c]	Type L_1 50	23	Type L_2 36	28	Type L_3 34	41	Type L_4 21	42
1		2.36	2.35	1.79	1.59	1.12	1.27	1.62	2.07	2.03	1.12
2		2.31	1.91	1.79	1.59	1.04	1.27	1.27	1.84	2.38	1.11
3		2.36	1.91	1.37	1.63	0.85	1.23	1.92	1.47	2.44	0.90
4		2.34	1.96	1.79	1.63	1.03	1.15	1.32	1.47	2.33	1.31
5		2.36	1.91	1.66	1.99	1.04	1.09	2.20	1.26	2.19	1.06
6		2.34	1.85	1.79	1.59	0.92	1.09	1.11	1.32	2.35	1.11
7		2.35	1.91	1.44	1.28	1.02	1.15	1.28	0.92	2.38	0.98
8		1.90	1.91	1.79	1.59	1.03	1.28	1.25	1.07	2.39	1.12
Orthogonal fitting		2.32	2.45	1.58	2.15	1.61	1.59	1.64	2.23	1.99	1.20
Standard		2.29	2.50	2.12	1.75	1.68	1.69	1.50	1.35	1.98	1.28

[a]Heavily stubbed

[b]Long-lived data

[c]Short-lived data

139

Table 12b. Expectancy-average service life summations, left modal data[a]

Panel member	No.:	Type L_0		Type L_1		Type L_2		Type L_3		Type L_4	
		10	27	50	23	36	28	34	41	21	42
9		1.86	2.03	1.39	1.52	1.08	0.86	0.62	0.99	1.80	1.37
10		1.86	2.03	1.51	1.37	1.38	0.99	0.87	0.99	1.52	1.16
11		1.86	2.03	1.32	1.31	1.00	1.06	1.00	1.06	2.05	1.34
12		2.00	2.03	1.39	1.37	1.13	1.05	0.85	1.17	1.58	1.22
13		2.00	2.03	1.27	1.37	1.07	0.84	0.85	1.11	2.07	1.16
14		1.96	2.03	1.27	1.37	1.07	1.06	0.81	1.06	1.56	1.34
15		2.02	2.03	1.32	1.37	0.92	0.96	0.73	1.29	1.52	1.16
16		2.02	2.03	1.20	1.37	1.11	1.18	0.81	1.31	1.25	1.34
Orthogonal fitting		2.26	2.20	1.79	1.41	1.91	1.58	1.56	1.21	1.67	1.34
Standard		1.98	2.31	1.78	1.65	1.62	1.30	1.07	1.24	1.28	1.31

[a]Light stubbing

Table 13a. Expectancy-average service life summations, right modal data[a]

Panel member No.:	Type R_1 15[b]	22[c]	Type R_2 13	18	Type R_3 8	6	Type R_4 4	46	Type R_5 37	26
1	2.42	2.27	1.99	1.20	2.21	1.77	1.53	1.47	1.71	2.08
2	2.35	1.91	2.26	1.40	2.22	1.65	1.59	1.64	1.76	1.90
3	2.42	2.28	1.84	1.02	2.21	1.49	1.91	1.40	1.71	1.81
4	2.43	2.28	2.23	1.08	2.24	1.79	1.91	1.69	2.12	2.08
5	1.90	2.30	2.32	1.20	1.87	1.82	1.91	1.42	1.84	2.40
6	2.42	1.78	2.21	0.82	2.22	1.79	1.53	1.42	1.84	2.42
7	2.37	2.28	2.08	1.00	2.10	1.82	1.90	1.21	2.59	1.65
8	1.90	2.27	1.99	1.54	1.90	2.15	1.59	1.11	1.81	2.06
Orthogonal fitting	2.34	2.06	3.07	2.09	2.34	2.20	1.46	2.06	1.88	1.74
Standard	2.02	2.16	1.86	1.43	1.66	1.53	1.38	1.53	1.68	1.35

[a] Heavily stubbed

[b] Long-lived data

[c] Short-lived data

Table 13b. Expectancy-average service life summations, right modal data[a]

Panel member	No.:	Type R_1 15	22	Type R_2 13	18	Type R_3 8	6	Type R_4 4	46	Type R_5 37	26
9		1.63	2.10	1.88	0.97	1.50	1.74	1.21	1.23	1.02	1.86
10		1.58	2.07	1.92	0.85	1.47	2.01	1.21	1.18	1.05	1.09
11		2.13	1.94	1.97	0.56	1.96	2.10	1.33	2.04	1.01	1.07
12		1.63	1.47	1.96	1.18	1.50	1.59	1.52	1.70	1.05	1.09
13		2.17	1.47	1.88	0.93	1.88	1.67	1.52	1.61	1.02	1.09
14		1.63	1.47	1.85	1.13	1.47	1.68	1.28	1.02	1.02	1.09
15		1.63	2.07	1.46	1.04	1.43	1.57	1.35	1.09	1.05	1.09
16		1.63	1.47	1.88	1.38	1.47	1.28	1.09	1.60	1.05	1.09
Orthogonal fitting		1.51	2.82	1.77	1.11	1.32	2.26	1.20	1.08	1.11	1.04
Standard		1.79	1.71	1.34	1.17	1.10	1.30	1.40	1.03	1.17	1.15

[a]Lightly stubbed

Table 14a. Expectancy-average service life summations, symmetrical data[a]

Panel member No.:	Type S_0 43[b]	47[c]	Type S_1 45	2	Type S_2 31	20	Type S_3 17	14	Type S_4 29	39
1	1.66	1.62	1.82	1.80	1.87	2.06	1.38	1.65	1.74	1.81
2	1.56	2.08	1.86	1.53	1.87	1.64	1.41	1.65	1.40	1.65
3	1.56	1.79	1.82	1.80	1.83	1.89	1.46	1.45	1.65	1.73
4	1.64	2.12	1.86	1.63	1.86	1.44	1.43	1.45	1.74	1.81
5	1.62	2.44	1.43	1.67	1.46	1.89	1.44	1.49	1.66	1.85
6	2.01	2.03	1.86	1.63	1.84	1.83	1.38	1.24	1.27	1.73
7	1.99	2.12	1.82	2.01	1.89	2.18	0.55	1.49	1.77	1.81
8	2.01	1.73	1.82	1.80	1.84	2.06	1.38	1.15	1.27	1.81
Orthogonal fitting	1.88	1.64	2.02	1.74	1.97	1.98	1.57	1.37	1.63	1.78
Standard	2.13	1.86	1.88	1.90	1.68	1.76	1.53	1.53	1.52	1.59

[a]Heavily stubbed

[b]Long-lived data

[c]Short-lived data

143

Table 14a. Expectancy-average service life summations, symmetrical data[a]

Panel member No.:	Type S_0 43	47	Type S_1 45	2	Type S_2 31	20	Type S_3 17	14	Type S_4 29	39
9	1.82	1.83	1.44	1.23	1.61	1.67	1.14	1.07	1.44	1.50
10	1.67	1.40	1.48	1.23	1.24	1.67	1.44	1.07	1.12	1.48
11	1.84	1.34	1.47	1.74	1.24	1.59	1.27	1.12	1.47	1.48
12	1.17	1.16	1.87	1.23	1.53	1.72	1.27	1.26	1.46	1.55
13	1.65	1.75	1.47	1.23	1.53	1.64	1.27	1.24	1.25	1.16
14	1.65	1.61	1.47	1.23	1.30	1.67	1.14	1.47	1.15	1.50
15	1.69	1.72	1.44	1.76	1.61	1.62	1.46	1.20	1.46	1.55
16	1.65	1.40	1.47	1.74	1.20	1.62	1.46	1.07	1.12	1.48
Orthogonal fitting	2.45	1.54	1.48	1.59	1.52	1.59	1.32	1.16	1.29	1.45
Standard	1.71	1.64	1.41	1.49	1.35	1.51	1.25	1.26	1.20	1.28

[a]Lightly stubbed

APPENDIX D: SAMPLE CALCULATIONS

Life table No. 42 has been chosen as the basis for the following illustrations. The complete, unsmoothed data are presented below in Table 15.

Table 15.   Original data for life table no. 42

Age interval years	Survival ratio	Survivors, beginning of interval, %
0 - ½	0.9996	100.00
½ - 1½	0.9954	99.96
1½ - 2½	0.9992	99.50
2½ - 3½	0.9916	99.41
3½ - 4½	0.9862	98.58
4½ - 5½	0.9680	97.23
5½ - 6½	0.9160	94.12
6½ - 7½	0.7828[a]	86.21
7½ - 8½	0.8189[b]	67.48[a]
8½ - 9½	0.7098	55.26[b]
9½ - 10½	0.5401	39.22
10½ - 11½	0.4009	21.19
11½ - 12½	0.4308	8.49
12½ - 13½	0.6451	3.66
14½ - 15½	1.0000	2.36
15½ - 16½	0.3131	2.36
16½ - 17½	0.4281	0.74
17½ - 18½	0.0000	0.31
18½ - 19½	--	0.00

[a]Terminal values for test data No. 42-C

[b]Terminal values for test data No. 42-S

Comparison Standards

Average life

The total service to be rendered by a property behaving as life table No. 42 is found by

$$\text{Total service} = 25\% + 3/4\ L_{\frac{1}{2}} + \text{Remaining terms in life table.}$$

Therefore, the average service life of the group is

$$\text{Average service life, } E = \frac{\text{Total service}}{100\%}$$

$$= \frac{\text{Sum of all life table terms} - (75\% + \frac{1}{4}L_{\frac{1}{2}})}{100\%}$$

$$= \frac{976.08 - (75 + 24.99)}{100\%}$$

$$= 8.76 \text{ years.}$$

This figure is rounded off to nine years.

Expectancy-average service life ratios

The terminal age for life table No. 42-S is $8\frac{1}{2}$ years. Expectancy-average life ratios are to be determined at equally spaced intervals from this age back to age zero, or at ages $2\frac{1}{2}$, $5\frac{1}{2}$, and $8\frac{1}{2}$. The expectancy at any age is expressed as

$$\text{Expectancy at age } x,\ E_x = \frac{\text{Total service remaining from age } x}{\text{Survivors at age } x}$$

$$= \frac{L_x \quad \text{Remaining terms from life table}}{L_x}$$

$$- \frac{1}{2}$$

Thus,

$$E/E_x, \text{ age } 2\tfrac{1}{2} = \frac{676.62}{(99.41)(9)} - \tfrac{1}{2}$$

$$= 0.70$$

$$E/E_x, \text{ age } 5\tfrac{1}{2} = \frac{381.40}{(94.12)(9)} - \tfrac{1}{2}$$

$$= 0.40$$

$$E/E_x, \text{ age } 8\tfrac{1}{2} = \frac{133.59}{(55.26)(9)} - \tfrac{1}{2}$$

$$= 0.21.$$

The standard is merely the sum of these three ratios, or

Expectancy-average service life summation standard, No. 42-S

$$= 0.70 + 0.40 + 0.21$$

$$= 1.31.$$

Iowa Type Curves--Test Curve No. 42-S

Average life

No additional calculations are needed since the average life is estimated directly by the panel member. For these data, panel member No. 9 chose an Iowa type S_3, 9 year average life.

Expectancy-average service life ratio

The expectancies are tabulated at most all ages for all Iowa type curves (15). Thus, the expectancies at ages $2\tfrac{1}{2}$, $5\tfrac{1}{2}$, and $8\tfrac{1}{2}$ are found to be 6.5 years, 3.8 years, and 2.0 years, respectively, for a type S_3, 9 year average life.

Therefore, panel member No. 9 estimated the expectancy-average service life summation as

$$\leq E_x/E = \frac{6.5 + 3.8 + 2.0}{9}$$

$$= 1.37.$$

Orthogonal Polynomials--Test Curve No. 42-S

Fitting procedure

This method is applied directly to the retirement ratios as given by the data. The survivor ratios as shown in Table 15 are, therefore, converted to retirement rates by subtraction from unity and listed in the first column of Table 16. Since the values used must be for equal increments in age, the factor for the $0-\frac{1}{2}$ age interval must be excluded.

Moving sums are then determined from these data as shown in the second, third, and fourth columns of Table 16. This was done by adding the column of ratios, but before each term was added to the next, the intermediate sum was recorded in the next column, i.e., $\leq_2$. The resulting second column was then similarly summed producing the third column, etc. The process was continued until enough sums were developed for a third degree solution. Each column total after the first provides the information for an additional degree.

Table 16. Retirement ratio summations, test data No. 42-S

Interval number	Retirement ratios			
	$\leq_1$	$\leq_2$	$\leq_3$	$\leq_4$
1	0.0046	0.0046	0.0046	0.0046
2	0.0008	0.0054	0.0100	0.0146
3	0.0084	0.0138	0.0238	0.0384
4	0.0138	0.0276	0.0514	0.0898
5	0.0320	0.0596	0.1110	0.2008
6	0.0840	0.1436	0.2546	0.4554
7	0.2172	0.3608	0.6154	1.0708
8	0.1811	0.5419	1.1573	2.2281
	0.5419[a]	1.1573	2.2281	4.1025

[a]Column total

The following calculations are completed with the
column totals, $\leq_1$, $\leq_2$, $\leq_3$, etc.:

$$a = \frac{1}{n} \leq_1 = \frac{0.5419}{8} = 0.0677$$

$$a' = a = 0.0677$$

$$b = \frac{2}{(n)(nH)} \leq_2 = \frac{2}{(8)(9)} (1.1573) = 0.0321$$

$$b' = a - b = 0.0677 - 0.0321 = 0.0356$$

$$c = \frac{6}{(n)(nH)(n+2)} \leq_3 = \frac{6}{(8)(9)(10)} (2.2281) = 0.0186$$

$$c' = a-3b+2c = 0.0677 - (3)(0.0321) + (2)(0.0186)$$
$$= 0.0084$$

$$d = \frac{24}{(n)(n+1)(n+2)(n+3)} \nleqslant 4 = \frac{24}{(8)(9)(10)(11)} \quad (4.1025)$$

$$= 0.0124$$

$$d' = a-6b+10c-5d = 0.0677 - (6)(0.0321) + (10)(0.0186)$$

$$- (5)(0.0124)$$

$$= -0.0016$$

where n is the number of age intervals considered.

At this point either the first, second, or third degree equations could be determined. Symbolically, these expressions may be shown as follows:

$$R = A + B\mathcal{E}_1 ;$$

$$R = A + B\mathcal{E}_1 + C\mathcal{E}_2 ;$$

$$R = A + B\mathcal{E}_1 + C\mathcal{E}_2 + D\mathcal{E}_3 .$$

where   R = retirement ratio at any age x

$\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3$ = functions of x in first, second, and third

degrees, respectively

A, B, C, D = constants determined by data.

The functions, $\mathcal{E}_1$, $\mathcal{E}_2$, $\mathcal{E}_3$, can be expressed in terms of the moments of the x distribution and the coefficients A, B, C, and D, are defined by factors previously calculated as

$$A = a'$$

$$B = - \frac{6}{n-1} b'$$

$$C = \frac{30}{(n-1)(n-2)} c'$$

$$D = \frac{140}{(n-1)(n-2)(n-3)} d'.$$

However, since a retirement ratio is wanted for each age interval, the solution of an equation would become quite cumbersome. There is an alternative approach which produces the entire series of values by a summation process once the appropriate degree of the equation has been selected.

The second degree form was chosen here as it was for most all life tables. It should be noted, however, that the third degree expression would be inappropriate for these data since the equation for the corresponding survivor curve would not reduce to zero. The reason for this is the sign of the term d'; its effect on the final expression may be seen by the relationship between d' and D.[a]

For a second degree equation three additional terms must be evaluated in order to develop the complete tabular solution for the smoothed ratios. These terms are as follows:

$$R_n = a' + 3b' + 5c'$$
$$= 0.0677 + (3)(0.0356) + (5)(0.0084)$$
$$= 0.2167;$$

$$\Delta R_n = - \frac{6}{n-1} (b' + 5c')$$
$$= - \frac{6}{8-1} \left[(0.0356) + (5)(0.0084) \right]$$
$$= -0.0667;$$

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[a] Supra, p. 65.

$$\Delta^2 R_n = \frac{60}{(n-1)(n-2)} \quad (c')$$

$$= \frac{60}{(8-1)(8-2)} \quad (0.0084)$$

$$= 0.0120.$$

A column of first differences is built up from the terminal value ΔR_n by successive additions of the constant second difference, $\Delta^2 R_n$. The smoothed value of the retirement ratios at the terminal point of the observed data, R_n, is then used as the basis for completing the solutions. This is done by adding successively the first differences mentioned above to the terminal value. The procedure is the same for the extrapolation of the data except that the signs of all operations are reversed. Table 17 illustrates this development.

Smoothed life table values can now be computed from the retirement ratios. It is to be noted that the negative ratios were assumed to have values of zero. Likewise, the ratios cannot exceed unity so any terms in this category were assumed to be one. For convenience, survivor ratios, computed from the corresponding retirement rates, were used in determining the life table values. The resulting data are shown in Table 18.

Table 17. Tabular solution for smoothed retirement ratios, No. 42-S

Interval number	Second difference	First difference	Retirement ratio
1		+0.0173	-0.0088
2		+0.0053	-0.0035
3		-0.0067	0.0032
4		-0.0187	0.0219
5		-0.0307	0.0526
6		-0.0427	0.0953
7		-0.0547	0.1500
8	+0.0120[a]	-0.0667[a]	0.2167[a]
9		-0.0787	0.2954
10		-0.0907	0.3861
11		-0.1027	0.4888
12		-0.1147	0.6035
13		-0.1267	0.7302
14		-0.1387	0.8689
15		-0.1507	1.0196

[a] Terminal values of stub data, No. 42-S

Table 18. Life table smoothed by orthogonal polynomials, No. 42-S

Age interval years	Survivor ratio	Survivors, beginning of interval, %
0 - ½	1.0000	100.00
½ - 1½	1.000	100.00
1½ - 2½	1.000	100.00
2½ - 3½	.9968	100.00
3½ - 4½	.9781	99.68
4½ - 5½	.9474	97.50
5½ - 6½	.9047	92.37
6½ - 7½	.8500	83.57
7½ - 8½	.7833	71.03
8½ - 9½	.7046	55.64
9½ - 10½	.6139	39.20
10½ - 11½	.5112	24.06
11½ - 12½	.3965	12.30
12½ - 13½	.2698	4.88
13½ - 14½	.1311	1.32
14½ - 15½	.0000	0.17
15½ - 16½	--	0.00

Average life

The average service life is found by a numerical integration of the smoothed life table giving the following results:

$$E = \frac{\text{Sum of life table terms} - (75\% + \frac{1}{4}L_{\overline{a}})}{100\%}$$

$$= \frac{137.57 - (75 + 25)}{100\%}$$

$$= 8.8172 \text{ years}$$

which is rounded to nine years.

Expectancy-average service life ratios

These ratios are determined at ages $2\frac{1}{2}$, $5\frac{1}{2}$, and $8\frac{1}{2}$. The computations are below:

$$E_x/E, \text{ age } 2\frac{1}{2} = \frac{681.72}{(100.00)(9)} - \frac{1}{2}$$

$$= 0.71$$

$$E_x/E, \text{ age } 5\frac{1}{2} = \frac{384.54}{(92.37)(9)} - \frac{1}{2}$$

$$= 0.41$$

$$E_x/E, \text{ age } 8\frac{1}{2} = \frac{137.57}{(55.64)(9)} - \frac{1}{2}$$

$$= 0.22$$

and, $\quad \sum E_x/E = 1.34.$

APPENDIX E: STATISTICAL ANALYSIS DATA

Table 19. Summarization of mean square values and corresponding degrees of freedom

Variation	Degrees freedom	Mean square values									
		Type L_0		Type L_1		Type L_2		Type L_3		Type L_4	
		E^d	$\leq E_x/E^d$	E	$\leq E_x/E$	E	$\leq E_x/E$	E	$\leq E_x/E$	E	$\leq E_x/E$
Correction for mean	1	203.1[a]	1.150	132.2	1.351	315.1	5.499	5.1	0.022	495.1	0.078
		45.6[b]	0.391	64.0	2.045	60.1	2.723	42.2	0.548	36.0	0.462
		56.2[c]	0.200	12.2	0.072	100.0	0.483	76.6	0.788	264.1	0.160
Total corrected	15	307.9	1.503	47.8	0.839	21.9	0.217	627.9	2.067	1033.9	1.320
		9.4	0.284	8.0	0.215	14.9	0.415	27.8	0.266	84.0	1.391
		287.8	0.787	53.8	0.554	16.0	0.239	624.4	2.649	686.9	1.084
Between groups	1	232.6	1.150	16.0	0.369	14.1	0.122	0.0	0.026	689.1	1.092
		1.6	0.245	0.2	0.126	5.1	0.202	16.0	0.073	64.0	0.766
		196.0	0.334	12.2	0.064	2.2	0.010	39.1	0.012	333.1	0.029
Within groups	14	5.4	0.025	2.3	0.034	0.6	0.007	44.9	0.146	24.6	0.016
		0.6	0.003	0.6	0.006	0.7	0.015	0.8	0.014	1.4	0.045
		6.6	0.032	3.0	0.035	1.0	0.016	41.8	0.188	25.3	0.075

[a]Heavily stubbed data

[b]Lightly stubbed data

[c]Stubbing effect data

[d]Comparison base abbreviations: E--average life; $\leq E_x/E$--expectancy--average service life summation

Table 19. (Continued)

Variation	Degrees freedom	Mean square values									
		Type R_1		Type R_2		Type R_3		Type R_4		Type R_5	
		E	$\leqslant E_x/E$	E	$\leqslant E_x/E$	E	$\leqslant E_x/E$	E	$\leqslant E_x/E$	E	$\leqslant E_x/E$
Correction for mean	1	240.2	0.286	182.2	0.001	2116.0	2.052	248.1	0.238	3422.2	3.553
		3.1	0.001	264.1	0.476	1463.1	3.168	342.2	0.735	0.1	0.042
		189.1	0.263	7.6	0.526	60.1	0.121	7.6	0.137	3393.1	4.368
Total corrected	15	353.8	0.916	341.8	1.679	580.0	0.573	726.9	1.370	3421.8	1.966
		71.9	1.160	268.9	2.426	2079.9	0.796	591.8	2.579	70.9	0.641
		513.9	2.478	305.4	1.050	4308.9	1.982	1955.4	6.211	3495.9	1.889
Between groups	1	144.0	0.240	169.0	1.113	56.2	0.170	495.1	0.860	225.0	0.837
		0.6	0.028	203.1	1.822	95.1	0.026	240.2	1.446	3.1	0.116
		162.6	0.432	1.6	0.087	297.4	0.064	1424.8	4.537	280.6	0.331
Within groups	14	15.0	0.048	12.3	0.040	37.4	0.029	16.6	0.036	228.3	0.081
		5.1	0.081	4.7	0.043	141.8	0.055	25.1	0.081	4.8	0.038
		25.1	0.146	21.7	0.069	286.5	0.137	37.9	0.120	229.7	0.111

Table 19. (Continued)

Variation	Degrees freedom	Type S_0		Type S_1		Type S_2		Type S_3		Type S_4	
		E	$\leq E_x/E$	E	$\leq E_x/E$	E	$\leq E_x/E$	E	$\leq E_x/E$	E	$\leq E_x/E$
Correction for mean	1	30.2	0.235	56.2	0.270	76.6	0.238	105.1	0.384	272.2	0.207
		1.6	0.013	3.1	0.006	25.0	0.156	20.2	0.001	81.0	0.339
		45.6	0.139	85.6	0.354	14.1	0.009	217.6	0.345	56.2	0.016
Total corrected	15	325.8	1.812	31.8	0.325	78.4	0.552	705.9	0.954	251.8	0.432
		59.4	0.732	35.9	0.763	27.0	0.262	63.8	0.325	137.0	0.321
		493.4	3.398	63.4	0.505	112.9	0.987	985.4	1.823	181.8	0.263
Between groups	1	196.0	1.020	2.2	0.021	7.6	0.001	189.1	0.081	110.2	0.081
		7.6	0.117	14.1	0.116	0.2	0.027	4.0	0.066	25.0	0.022
		280.6	1.829	27.6	0.028	10.6	0.372	248.1	0.294	30.2	0.019
Within groups	14	9.3	0.057	2.1	0.022	5.1	0.039	36.9	0.062	10.1	0.025
		3.7	0.044	1.6	0.046	1.9	0.017	4.3	0.018	8.0	0.021
		15.2	0.112	2.6	0.033	7.3	0.044	52.7	0.109	10.8	0.018

Mean square values